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From the Editor

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PREFACE

Cantor's set theory combined with the development of formal logic changed the foundations of mathematics forever. In the late 1920s, David Hilbert, who was one of the most influential mathematicians of his time, was advancing a program the aim of which was to establish once and for all the validity of infinitistic methods that proved to be so powerful in various areas of classical mathematics. Hilbert's program suffered a fatal blow when Kurt Gödel announced his incompleteness theorems. Some, including John von Neumann, instantly grasped the importance of Gödel's results, but it took the mathematical community much longer to realize what the results meant for the foundations of mathematics and for mathematical practice. Craig Smoryński tells an interesting story about it in *Hilbert's Programme* (1988).

It was only in 1950's, after seminal work in proof theory and computability theory, by Ackerman, Bernays, Church, Gentzen, Hilbert, Kleene, Post, Turing, and many others, that one could say with confidence that we now know how to formalize mathematics. Equipped with the new conceptual framework, certain foundational issues became approachable, and one could hope to establish results about them with mathematical precision. In this vein, John Lucas in the 1960s and later Roger Penrose in the 1990s came up with arguments, based on Gödel's theorems, to show that mathematics as human activity cannot be reduced to a single algorithmic procedure, or, more poetically, that human minds are not machines. Prior to that, Gödel himself, in his Gibbs lecture in 1951, formulated and argued for what is now known as Gödel's disjunction:

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> Either the human mathematical mind cannot be captured by an algorithm, or there are absolutely undecidable problems.

Gödel believed that both disjuncts were true, and was convinced that a rigorous confirmation of the disjunction could be given, but he could not see a way to do it for either of the disjuncts. Lucas and Penrose argued that the first disjunct holds. Recently, in a series of articles, Peter Koelner provided a formal framework to validate Gödel's disjunction (2016; 2018a; 2018b); but, after a thorough analysis, the arguments of Lucas and Penrose have been rejected by the logic community. Stanisław Krajewski's essay in this collection provides a detailed analysis of Lucas' proof and two proofs given by Penrose. It was the initiative of the editors of *Semiotic Studies* to invite mathematicians and philosophers to respond to Krajewski's essay and to comment on related issues from todays perspective.

While not much can be added the logical analysis of the arguments of Lucas and Penrose, the question of mechanization of mathematics gives rise to a discussion that touches upon central problems in the philosophy of mathematics. As computer-assisted proofs become routine, it is also relevant to current mathematical practice. In the June/July 2018 issue of the Notices of the American Mathematical Society, Jeremy Avigad gives a survey of recent advances in automated theorem proving, and in the conclusion he writes:

> The history of mathematics is a history of doing whatever it takes to extend our cognitive reach, and designing concepts and methods that augment our capacities to understand. The computer is nothing more than a tool in that respect, but it is one that fundamentally expands the range of structures we can discover and the kinds of truths we can reliably come to know. This is as exciting a time as any in the history of mathematics, and even though we can only speculate as to what the future will bring, it should be clear that the technologies before us are well worth exploring. (2018)

The role of proof in mathematics is not just to discover mathematical truths, but rather to provide insights into why this or that particular statement is true. Surely, such insights cannot be provided by a machine that only spits out true mathematical statements one by one; most of them are simply uninteresting. However, it would be a mistake to underestimate what machines can actually do. Stephen Wolfram, the founder of *Mathematica*, discusses this in his blog:

> At some level I think it's a quirk of history that proofs are typically today presented for humans to understand, while programs are usually just thought of as things for computers to run. Why has this happened? Well, at least in the past, proofs could really only be represented in essentially textual form—so if they were going to be used, it would have to be by humans. But programs have essentially always been written in some form of computer language. And for the longest time, that language tended to be set up to map fairly directly onto the low-level operations of the computer—which meant that it was readily "understandable" by the computer, but not necessarily by humans. But as it happens, one of the main goals of

> my own efforts over the past several decades has been to change this—and to develop in the Wolfram Language a true "computational communication language" in which computational ideas can be communicated in a way that is readily understandable to both computers and humans. (2018)

The articles in this issue can be divided into three groups. Krajewski's article, Yong Cheng's contribution, and a short note by Rudy Rucker, provide detailed mathematical analysis of Lucas-Penrose type arguments. In the second group, with articles by Arnon Avron, Stepan Holub, Panu Raaikiainen, and Albert Visser, the authors discuss the status and various methodological and technical problems of the anti-mechanist arguments. In essence: what does the problem of "minds vs. machines" really mean, and how can it, and how should it, be formulated? Moreover: How to evaluate the merit of arguments that mix formal mathematics and philosophical considerations? The third group consists of the articles that, while including issues from the other two groups, concentrate of more specific themes: an analysis of Georg Kreisel's observation that it does not logically follow from the fact that a formal system is subject to the second Gödel incompleteness theorems that there are absolutely no means available to prove its consistency (Jeff Buechner); Per Martin-Löf's proof that there are no absolute unknowables in constructive mathematics (V. Alexis Peluce); diagonal arguments and Chomsky's approach to linguistic competence as contrasted with arithmetic competence (David Kashtan); and the role in the anti-mechanist arguments of difficulties in capturing the nature of natural numbers in formal systems (Paula Quinon).

All articles in this issue, directly or indirectly, address the limits of mathematical knowledge. While we have a precise definition of provability in formal systems, the question of what is knowable is vague. In a series of recent papers, Peter Koelner approached this problem, by formalizing aspects of mathematical truth and knowability in a way that allows him to give a rigorous argument validating Gödel's disjunction. This theme is taken up in Yong Cheng's article in this issue.

Finally, Wilfred Sieg's article gives a historical account of the seminal contributions of Gödel and Turing that made possible all later developments partially described in this issue.

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