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Article

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LIMITATIONS OF FORMAL (LOGICAL) SEMANTICS

Summary: According to the received view formal semantics applies to natural (ordinary) language to some extent only. It is so because natural language is inherently indefinite, in particular, its expressions are ambiguous, vague and admits departures from syntactic rule. Moreover, intensional contexts occur in ordinary language—it results in limitations of the principle of compositionality. The ordinary conversation appeals to various principles, for instance, Grice's maxims which exceed logical formalism. Thus, ordinary language cannot be fully formalized. On the other hand, if L is a formal language, its metalanguage ML, must be partially informal—for instance, it contains, terms of ordinary mathematics, especially set theory. Even, if, for instance, due to the technique of aritmetization, ML can be represented in L, such a representation is only local. In fact, this view can be derived from some Tarski's remarks on the role played by natural language. It is usually assumed that the universality of natural language, is the source of troubles associated with antinomies. It is so this circumstance requires a solution, for example by distinguishing levels of language. However, even if antinomies are excluded, what is informal is prior with respect to what is formal. It shows that formal semantics has limitations even with respect to formalized languages.

Keywords: language, metalanguage, model, logic, syntax, semantics.

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Although the term "semantics" occurs in many contexts, it always concerns signs. In my further remarks, I will focus on linguistic signs. At first, semantics appeared as a part of linguistics and changes of meaning were its subject. However, philosophers, in particular, logicians, very quickly became interested in semantics.[1] In the Polish tradition semantics belongs to logic *sensu largo*, next to logic *sensu stricto* (formal logic) and methodology of science. Disregarding this last part of logic, the problem arose of how semantics and formal logic are mutually related. The situation was complicated due to the introduction of (see Morris, 1938; I neglect earlier terminological proposals) the term "semiotics" denoting the general theory of signs and its division into syntax, semantics and pragmatics. Charles Morris was influenced by American pragmatism, particularly the ideas of Charles Sanders Peirce. His (Morris's) tripartite division follows the distinction of three parts of semiotic situation (semiosis), that is, such that signs function in it. Firstly, signs are related to other signs, refer to something and are used by someone—an interpreter. Summing up, signs remain in formal relations (with respect to forms) with other signs, denote something and are signs for someone—this third aspect consists in that if S is a sign, and U—its user, S expresses contents existing in U. It always happens, when we consider relations occurring between users of a language and expressions employed by them. For example, if we say that a person P asserts or accepts a sentence A, we point at a relation between the person in question and an assertion or acceptance. Supposing, believing, doubting, questioning or demanding are further examples of pragmatic situations, frequently called—propositional attitudes.

The terminological complication is additionally made more complicated by the fact that the labels "semiotics" and "semantics" function almost as synonyms. Accordingly, we have names "semantics in a broad sense" and "semantics in a narrow sense". The latter focuses on relations between expressions and what they refer to—the concept of truth (in Alfred Tarski's sense) and denotation or designation are typical semantic notions in this sense. On the other hand semantics is also understood as the theory of meaning, not in the traditional linguistic sense, that is, as registering changes of meanings of expressions, but in a more philosophical enter-

[1] My remarks on semantics are very general and do not go beyond very elementary matters. A wider account can be found in (Pelc, 1982). I employ my considerations in (Woleński, 2020, Chapter 6).

prise, for instance, when investigations concern such categories as analyticity, synonymy or meaningfulness. If semantics is declared as a study not only of reference (more generally, referential relations), but also meaning, the characterization of this latter concept is indispensable. Since I cannot enter here even into a very general typology of meaning-theories, I limit myself to a remark that meaning is the relation which decides that we understand expressions. This statement locates the concept of meaning somewhere on the borderline of semantics as the theory of reference (an expression is understood, provided that it is known to what it refers to) and pragmatics (expressions are always comprehended by an interpreter).

Rudolf Carnap offered the following explanation:

> If in investigations explicit reference is made to the speaker, or to put it in more general terms, tie the user toa language, then we assign it to the field of p r a g m a t i c s . [...] If we abstract from the user of the language and analyze only the expressions and their designata, we are in the field of s e m a n t i c s . And, if finally, we also abstract from the designata and analyze only the relations between the expressions, we are in (logical) s y n t a x . The whole science of language, consisting of the three parts mentioned, is called s e m i o t i c s . (1939, p. 146; emphasis in the original—J.W.)

According to Williard Quine:

> When we cleavage between meaning and reference is properly heeded [...], the problems of what is loosely called semantics become separated into two provinces so fundamentally distinct as not to deserve a joint appellation at all. They may be called the t h e o r y o f m e a n i n g and the t h e o r y o f r e f e r e n c e . "Semantics" would be a good name for the theory of meaning, were it not for the fact that some of the best works in so-called semantics, notably, Tarski's, belong to the theory of reference. The main concepts in the theory of meaning, apart from meaning itself, are s y n o n y m y (or sameness of meaning), s i g n i f i c a n c e (or possession of meaning), and a n a l y t i c i t y (or truth in virtue of meaning). Another is e n t a i l m e n t , or analyticity of the conditional. The main concepts in the theory of reference are n a m i n g , t r u t h , d e n o t a t i o n (or truth-of), and extension. Another is notion of v a l u e s of variables. (1953, p. 130; emphasis in the original—J.W.)

This view sees a sharp contrast between theory of meaning and theory of reference and reserves the name "semantics" to the latter.

What Tarski himself understood by semantics is indicated in the following passages:

> (a) [...] we attempted to go further and to construct [...] definitions and concepts belonging to semantics of a language—i.e. such concepts as satisfaction, denoting, truth, definability, and so on. A characteristic feature of the semantical concepts is that they give expression to certain relations between the expressions of language and the objects about which these expressions speak, or that by means of such relations they characterize certain classes of expressions or other objects. We could also say (making use of the *suppositiomaterialis*) that concepts serve to set up the correlation between names of expressions and the expressions of themselves. (Tarski, 1933, p. 252)

> (b) The word "semantics" is used here in a narrower sense than usual. We shall understand by semantics the totality of considerations concerning those concepts which, roughly speaking, express certain connexions between the expressions of a language and the objects and states of affairs referred to by these expressions. As typical examples of semantical concepts we may mention the concepts of d e n o t a t i o n , s a t i s f a c t i o n , and d e f i n i t i o n , [...]. The concept of t r u t h —and this is not commonly recognized—is alsoto be included here, at least in its classical interpretation. (Tarski, 1936, p. 401)

> (c) the study of the relations between models of formal systems and the syntactical properties of these systems (in other words, the semantics of formal systems). (Tarski, 1954, p. 714)

Although Quine offered serious argument for his account of the theory of meaning and the theory of reference, it is difficult to agree that the contexts "S means that m" and "S refers to o" are mutually separated. Tarski saw this in the following way:

> It remains perhaps to add that we are not interested here in "formal" languages and sciences in one special sense of the word "formal", namely sciences to the signs and expressions of which no material sense is attached. For such sciences the problem here discussed [the problem of truth—J.W.] has no relevance, it is not even meaningful. We shall always ascribe quite concrete and, for us, intelligible meanings to the signs which occur in the languages we shall consider. The expressions which we call sentences still remain sentences after the signs which occur in them have been translated into colloquial language. The sentences which are distinguished as axioms seem to us to be materially true, and in choosing rules of inference we are

> always guided by the principle that when such rules are applied to true sentences the sentences obtained by their use should also be true. (Tarski, 1933, p.166/167)

Although in his writings Tarski rather avoided answering about what meaning is, on the other hand, he maintained that semantic problems can be considered only with respect to languages with expression equipped with "quite concrete and, for us, intelligible meanings".

The circumstances pointed out above concern various aspects of adding the adjective "logical" to the nouns "semiotics", "syntax", "semantics" or "pragmatics". The sequence <syntax, semantics, pragmatics> is related to a passing from simpler to more complex matters. As a matter of fact, the syntactic description of a language is the simplest, because it takes into account exclusively the material side of signs and their corresponding relations, for instance, that the sign S is contained in a sign S'. The semantic treatment has to consider relations of signs to their objectual (principally, extralinguistic) correlations and, finally, the pragmatic aspects require an appeal to the interpreters (users) of expressions. Accordingly, syntax is relatively simple (in this respect, it is analogous to grammatical syntax), logical semantics touches "logic" (quotes intended), but logical pragmatics appears as the most complicated part of semiotics. Consequently, the term "logical semiotics" (or "logical semantics in a wider sense") refers to considerations consisting of syntax, semantics and pragmatics, each qualifies as logical.

Yet we can observe that more proper is the succession <pragmatics, semantics, syntax>, because signs, as was earlier noticed, always function as items referring to something and used by someone (an interpreter). Hence, every language possesses an irremovable pragmatic factor. Morris (1938) presented particular parts of semiotics by the sequence <semantics, pragmatics, syntax>, but it was rather an accidental convention. Carnap (1939) offered the already mentioned succession <pragmatics, semantics, syntax>. It has (Carnap probably considered it as so obvious that he did not mention it) a simple justification. Semantics has arisen by abstraction from the pragmatic aspect, and syntax omits referential relations. Perhaps more important is that a natural way of defining signs consists in pointing out that it is an object possessing meaning. Independently of complications associated with any analysis of meaning this category, clearly distinctive for sign, has a semiotic character, not only semantic or syntactic, but just pragmatic. This decides that pragmatics

(or its common terrain with semantics) is the proper place for a theory of meaning. If we say that, for example, a word is a bearer of a sign S, this fact, according to our main presumption, consists in possessing a meaning by S, independently whether meanings are conceived as mental states, ideal entity, etc. Consequently, the concept of meaning is assumed just in the definition of the sign-bearer.

On the other hand, it would be difficult to agree with logical empiricism that the concept of meaning can be exclusively explained on the base natural (material, physical) properties of signs, that is, their syntactic attributes. Suppose that I observe a combination of sand-pieces forming the word "horse" on a beach. Until I know whether it is a result of "cooperation" of water and wind, or written by a man or woman, I cannot decide whether it is a sign or not. That semantics is not reducible to syntax can be regarded as a canon of contemporary analytic philosophy. The relation between pragmatics to logical semantics appears less explicitly. The basic difficulty in this respect stems, to stress this point once again, from a proper location of the concept of meaning. If it belongs to pragmatics, elimination of pragmatic coordinates, in particular, propositional attitudes (or other pragmatic parameters) must be considered as dubious, but if the concept of meaning is placed within semantics, it is possible to try various formalizations by means of so-called intensional logics (see Carnap, 1947 for early attempts of intensionality analysis; van Benthem, 2002 for formal logical constructions; Parsons, 2002 for a survey of semantic constructions and elimination of such concepts as intensions, senses, meanings, etc. to referential relations).

I will not continue a further analysis of various possible settings or terminological proposals. Let us agree to employ the label "logical semantics" as referring to considerations about language which applies the apparatus of formal logic to analysis of referential relations of expressions, but assuming that they have meaning. Accordingly, logical semantics can be also regarded as formal semantics. Here appears a crucial question "Which languages admit applications of logical (formal) semantics?" In general, we have two standpoints as answers to this question, namely formalism and anti-formalism. To begin with the latter, Wittgenstein and representatives of so-called ordinary language philosophy (in particular, John L. Austin, Gilbert Ryle, Peter Strawson) entirely rejected employing for-

mal-logical methods to analysis of languages, particularly common speech.[2] In formalism, we can distinguish two versions of this view, namely radical and moderate. The former view maintains that formal semantics is possible only for formalized languages, but the latter, assumes the so-called Montague thesis (Montague, 1970; see Cann, 1993 for an extensive treatment)—that there is no principal difference between formalized (artificial) languages and natural (ordinary) languages. In particular, according to Richard Montague, every natural language is an interpretation of a formal language, and, if so, its (natural) logical analysis is fully justified. At the beginning of his career, Tarski accepted radical formalism, but he (1944) modified it to some extent. He admitted so-called languages with specified structure—they are local, that is, they do not constitute ordinary language in its integrity. Tarski always stressed (see above) that formalized languages are interpreted, that is, their expressions have meanings. On the other hand, he did not accept the Montague thesis, because, according to his view, formalization of an ordinary language always changes its character.[3]

Due to controversies around the Montague thesis we have a broad and narrow understanding of formal semantics. Under the former, it applies to every language (perhaps only to some extent in the case of ordinary language), but formalized languages (for simplicity, I omit languages with specified structure) become its proper scope under the latter approach. For closer account of this issue, I appeal to four contrasts

(A) natural–artificial;

(B) informal–formal;

(C) non-formalized–formalized;

(D) interpreted–non-interpreted.

Although the above distinctions cross over each other, it is profitable to introduce them and briefly characterize them. Natural (ordinary) language functions in our daily life and is accessible to everybody. It is con-

[2] I do not enter into a closer characterization of this orientation, although some anti-formalistic arguments will be brought up below.

[3] Jaakko Hintikka told me in a private conversation that Tarski explicitly criticized Montague's thesis. It happened in discussions—Tarski's writings contain nothing of this question.

trasted with artificial languages constructed for special tasks, for instance, logical symbolism, Morse's alphabet, binary code, chemical symbolism, etc. Such languages are formed in order to replace ordinary parlance in explicitly prescribed situations. They always have a well-defined structure, their syntax is regular and recursive (except admitting expressions of an infinite length, but it is a purely theoretical case), they do not need to be sets of sentences and usually satisfy the compositionality principle, that is, the rule that compound expressions are functions (in the mathematical sense) of their sub-expressions, provided with what we do with these sentences—for instance, chemical symbolism is not sentential and not compositional. Formal languages are constructed and then described according to strict rules, independently of the meanings or contents of expressions in question. Such rules can be identified with syntactic ones, that is, associated with form. On the other hand, informal languages require descriptions taking into account meanings (senses, contents) of expressions. A non-interpreted language has no interpretation—an interpreted language is such that it possesses a given interpretation via an interpretative rule. Finally, a formalized language arises as a product of formalization, but non-formalized language is not a result of formalization.

Limitations of formal semantics (at the moment I understand it as an analysis of signs via tools of formal logic) with respect to ordinary language are known and stressed for a long time. One of the reasons for scepticism in questions is (Kisielewicz, 2017) ambiguity, vagueness, homonymy, synonymy, amphibology, contextual dependency, etc. "Counterlogicality" (relatively to formal logic) of these properties of ordinary speech was particularly stressed by the ordinary language philosophy (the Oxonian School) and regarded as the circumstance which decided the lack of adequacy of formal semantics with respect to common parlance. Tarski (1933) pointed out that although it is possible to prepare ordinary languages toward well-defined logical artefacts, such a treatment deprives ordinary language of its naturalness as its essential feature and will result in converting it into an artificial system. Tarski also stressed that colloquial language is universal—it is its virtue, because if we want to say something, we always can do that in this language. Alterntively, the universality in question leads to semantic antinomies, because if **L** is a natural language, its metalanguage is its part. Logicians consider the properties of colloquial speech mentioned in the former paragraph as defects, requiring at least partial improvement, for instance, in the form of regulative definitions liquidating vagueness, ambiguities or other means toward

normalization of inaccuracies in question (see Trzęsicki, 2017 for an extensive treatment of these questions).

However, the foregoing observations require some corrections. It is true that so-called inaccuracies of ordinary language impede human communication on many occasions, because they can result in misunderstanding stemming from ambiguity or vagueness, but, on the other hand, they basically reduce the number of words indispensable for conversations. It is easy to imagine enormous troubles in transmitting information, provided that instead of the adjectives "tall", "short" and "medium height", we would need separate words for all possible cases of human height. If the mother says to her son "Bring me the picture of the castle in Kraków", she does not need to add "But remember that I am not speaking about hockey castle in Kraków", because the contexts of her request makes clear what is going on. Consequently, such circumstances as ambiguity and vagueness can act as benefits of communications, because they contribute to language-economy, although it results in troubles sometimes. If precision is required, for instance, in a legal text, we can always use definitions or other means of improving quality of our communication. Furthermore, although ordinary language does not precisely distinguish **L** and **ML**, this distinction is present in another one, namely *oratio recta* and *oratio obliqua* (the latter is intentional) and means to mark quotation. Finally, as Donald Davidson (1967) observed nobody uses the entire natural language, but always limits oneself to some part of it. Accordingly, this fragment can be logically improved, if it is justified for some reasons, e.g. for doing automatic translations or for resolving doubts language-users have by participating in an exchange of information,

The relation of ordinary language (more properly, its selected fragments) and its formalized version can be compared to that holding between theoretical physical models and corresponding "pieces" reality. One could say that laws of theoretical physics do not apply to the real world, because there is no absolute vacuum or mass of a body is not concentrated in one single point., etc. That all is true, but, in spite of these "theoretical" facts, mechanics (as a part of theoretical physics) is employed in practical statics and dynamics, laws of thermodynamics—in projecting equipment that secure temperature, hydrodynamics in construction of ships, etc. Since theoretical models are approximations of reality, these and similar applications are possible and legitimate. If this picture is correct, so-called defects of colloquial language have a deep cognitive importance and although they result in limitation of formal semantics,

should not be considered as circumstances testing its total non-applicability to analysis of daily parlance. Arguments invoked in this paragraph justify Tarski's already mentioned view (I recall that it was a revision of his earlier position) that a strict logical analysis can be performed not only with respect to fully formalized languages but also—to linguistic systems with a specified structure. Perhaps the Montague thesis should be weakened to the assertion that if we take into account an arbitrary portion of ordinary language, it always can be approximated (idealized) by a formal system. Consequently, some features of a formalized fragment disappear in the process of idealization, that is, considering a theoretical model (a formal system) as an admissible approximation of ordinary language. Consequently, limitations of formal semantics as applied to natural languages do not appear so fundamental as anti-formalists argue. It seems that they frequently confuse language in daily actions, that is a collection of concrete speech acts, and language as a product (a set of expressions) subjected to investigations by various methods, including formal-logical ones.

Formal semantics assumes formal logic or, in other words, the latter is a part of the former. It means that i.a. various logical operations have to be semantically legitimate. The concept of logical entailment (consequence) provides perhaps the best example. This notion can be defined syntactically, that is as the relation $X \vdash B$—a sentence B is deduced from the set X of sentences if and only if B is obtainable (provable) from X via inferential rules given in advance, for instance, we say that B follows inferentially from the set $\{A, A \Rightarrow B\}$ according to the detachment rule. On the other hand, we say that B follows semantically from X (symbolically, $X \models B$), if it is excluded that sentences belonging to X are true, but B—false. One can consider these definitions as an example of syntactic-semantic parallelism. Yet not every operation performed in practice has such legitimacy. The issue does not concern logical errors or various rhetorical games, but situations which have some rational justification from the point of view of interpersonal exchange of thought via linguistic expressions. This is a subject of the famous theory of so-called conversational maxims, formulated by Grice (1975). According to this idea, participants of an effective communication must mutually cooperate. This means that they should preserve some principles (maxims), in particular (I use a different terminology than Grice): (I) the truth–maxim (do not employ sentences, if you know that they are false); (II) the information-maxim (provide adequate information that is required by a given conver-

sation); (III) the substantiality-maxim (keep the topic of the conversation in question); (IV) the understanding-maxim (say clearly).

All of Grice's maxims are practical in their character, they formulate conditions of rational conversation. Additionally, they have an ethical dimension because if the task of conversation consists in the exchange of justified information, false sentences, or such that their correctness can be questioned, should be avoided, and transmitted information should not be too narrow or too wide so far as participants should be substantiated and provided for in a precise manner. Grice called his maxims implicatures, that is, as something which is semantically implied (in a wide sense) by using expressions, particularly sentences. The word "implicature" immediately brings associations with logic and logical entailment. However, it is impossible to reduce implicatures to logical consequence. Consider (I) and (II). The first recommends that falsehoods should not be sued. It could be eventually justified by appealing to a logical principle that a false sentence implies every sentence. Nevertheless, even if we say that a participant of a conversation can imply everything from a falsehood, it will not deduce arbitrary statements from such a premise. He or she will rather think (Grice himself strongly stressed this point) about intentions of his or her interlocutor or deliberated whether the falsehood in question perhaps contains a grain of truth or not. If the issue concerns (II), assume that someone answers to the question "What day of the week is today?" by saying "Today is Thursday or Friday", provided that he or she knows that the first eventuality holds (the question is stated on Thursday). Although the answer is true, it will be considered by the hearer as an expression of ignorance on the side of the questioned person. Yet this explanation is not correct, because the latter person just knows that today is Thursday. The maxim (III) requires a maximally true answer, but not—partial. The questioning person expects such an answer or "I do not know", if the questioned person does not actually know. One could say that if A is a correct answer to a given question, but the questioned person uses a sentence B such that B logically follows from A, but is not equivalent to it, B violates (II). Thus, although logic certainly touches various aspects of Grice's maxims, it neglects their pragmatic dimension, which is fundamental concerning their regulative role in conversational performances. We have made various attempts to construct a formal pragmatics (see Martin, 1959 for example), but all hitherto accessible evidence points out that limitations of formal semantics in analysis of

conversational contexts are fundamental just because irremovable pragmatic elements are present in using language.

I move on now to limitations of formal semantics with respect to languages inherently associated with logic. At this point, one should observe some relativity associated with the understanding of names "formal language" and "formalized language". Assuming that we have to do arithmetic of natural numbers in its shape known from mathematical practice. This theory is axiomatic, expressed in a "mixed" language containing arithmetical expressions as well as ordinary words—the latter are accustomed to expressing properties of natural numbers and relations holding between them. After formalization, we obtain a language, which can be qualified as formal. Clearly, it is assumed that the expression of the resulting formal system inherits the meaning (sense) of their counterparts before formalization. Let us consider a language $\mathbf{L}^*$ in which the symbol * functions as the only primitive. Moreover, we have only the one rule of forming new expressions of our language:

(R) if $\mathbf{E} \in \mathbf{L}^*$, then $\mathbf{E}^* \in \mathbf{L}^*$.

Due to these conventions, we conclude that $* \in \mathbf{L}^*$, $** \in \mathbf{L}^*$, $*** \in \mathbf{L}^*$, etc. Until now, nothing is known about the meaning of the expression *—in particular, we do not know whether it is a propositional symbol or not. Although the expressions "if, then" (we can admit that its meaning is established in propositional calculus) and $\in$ (the symbol of set-theoretical membership) appeared in the description of $\mathbf{L}^*$, they do not influence the meaning of the inscription *. Yet we can speak about formal language in the case of the language of arithmetic as well as in the case of $\mathbf{L}^*$constructed *ad hoc.* In both cases, if we have a language $\mathbf{L}$, its description is done in a suitable **ML**. This description covers syntax (a definition of an expression and rules of constructing compound expressions from simpler ones) As I noticed earlier the relation between $\mathbf{L}$ and **ML** is just as crucial for many problems of formal semantics, including its limitations.

We can think about $\mathbf{L}^*$ as the set of terms of English, the star *refers to a term consisting of a single word, but the rule (R) establishes that concatenation of simple terms is also a term. This explanation introduces an interpretation (this notion appeared earlier but only implicitly) of $\mathbf{L}^*$. The contemporary approach to semantic interpretation is closely associat-

ed with model-theoretic semantics. Assume that **L** is a language, that is a set of sentences. Let $\mathbf{X} \subset \mathbf{L}$, where **X** is consistent.[4] A model of **X** is a structure **M**, such that sentences belonging to **X** are true.[5] Intuitively, the truth of a given sentence A depends on the understanding of constituents of this sentence. Every model can be presented as an object $<\mathbf{U}, \mathbf{P_1}, \mathbf{P_2}, \mathbf{P_3}, ...>$, where **U** is a non-empty set of objects, but $\mathbf{P_1}$, $\mathbf{P_2}$, $\mathbf{P_3}$, ... are subsets of **U**. This description is associated with considering formulas of **L** as constructed from proper (individual) names, individual variables, predicates (predicate-letters) and logical constants. We assume that meanings of elements of this last catalogue (propositional connectives, quantifiers, identity) are established by logic. What about extralogical expressions? If a is a proper name or variable, its value in **M** is an object belonging to **M**; if P_i is a predicate it refers to the set $\mathbf{P_1}$. For instance, the sentence "Kraków is a city" expresses that Kraków is the value of the term "Kraków", but the set of cities is denoted by the predicate "is a city". Accordingly, the interpretation of the sentence in question can be formally accounted for by the expression "Kraków $\in$ **City**", where the word written in bold letters refers to the set of cities. This also means that that we apply set theory in the metalanguage, because the symbol $\in$ belongs to this theory. In general, an interpretation of expressions of a language **L** is a function, which maps its elements into their values, that is, objects from **M**. We can additionally say that interpretations determine that some sentences are true, but others false.[6]

Now the problem arises whether interpretations in the above sense ascribe intuitive meanings to expressions of a given language **L**? As I have

[4] **L** in its integrity is inconsistent, because for any sentence $A \in \mathbf{L}$, contains a negation.

[5] Model-theoretic semantics is based on the semantic theory of truth, introduced by Tarski (Tarski, 1933; see Woleński, 2020 for an extensive analysis of this theory). For simplicity, I neglect the situation of having many models by **X**. I return to this problem in connexion with non-standard models. I also ignore the distinction of first-order and higher-order logic, although intuitions and examples concern the former. My exposition is very simplified, but, I hope, it does not lead to confusion.

[6] According to one of the most fundamental metalogical results (the Gödel-Malcev completeness theorem), a set of sentences is consistent if and only if it has a model. Note that inconsistent sets of sentences have interpretations, though they possess no models.

already noticed, syntax of formal languages satisfies the principle of compositionality, that is, any compound sentence is a function of its simpler constituents. Since syntax and semantics should be parallel, this postulate motivates the compositionality (extensionality) of semantics. On the other hand, many reasons justify a sceptical attitude toward the universality of the compositionality of semantics. In particular, many colloquial contexts, including those typically analyzed via using logic, for instance, epistemic modalities ("I know that A"; or erotetic "I ask whether A") are non-extensional, etc.—I based this question above on the remarks on intensional logic. Since, to say that once again, prospects for overcoming difficulties in logical analysis of intensionality are problematic, we encounter here explicit limitations of formal semantics.

If we recognize semantics of classical calculus of sentences and predicates as a paradigm, we must conclude that model theory for intensional logics requires rather complicated tools such that their compositionality is dubious.[7] Independently of the problem of compositionality, model-theoretic semantics ignores the distinction, a fundamental one, of intension (meaning, content) and extension (reference, denotation, scope). It is particularly seen in the case or relation between predicates and sets as values of the former. It was observed for a long time that, for example, two expressions "the largest city on the Vistula river" and "the capital of Poland" have different intensions, but the same extension, that is Warsaw. In the case of predicates properties are considered as their intensions and sets—their extensions. Accordingly, the identity of meanings guarantees the identity of extensions, but not reversely—in the case of predicates, the identity of properties is sufficient for the identity of scopes. For instance, the proposition "Warsaw is the capital of Poland" does not entail the proposition that Warsaw is the largest city in Poland. Consequently, something more than information of extensions is necessary for making the model-theoretic semantics workable. And if we say that semantics is also occupied by senses, the answer is simple: to consider an interpretation of a formalized language $\mathbf{L}$ and its connection with a model $\mathbf{M}$, we need to know not only extensions of expressions but also their intensions. Although we can suppose, to return to one of the earlier examples, that the

[7] It does not mean that the construction satisfying the principle of Compositionality is of small importance or non-interesting. Introducing this contrast, I have in mind related differences, which are important in the present context.

interpretation of the sentence "Kraków is a city" employs set theory relatively to the relation expressed to the symbol $\in$, but this supposition is mediated by corresponding senses.

How to deal with this question, provided that if E is an expression of **L**, then its semantic analysis is limited to its value in **M**? The only reasonable solution to this problem which comes to mind, consists in also taking into account information about the meaning of E—the information in question is present in **ML**. It seems that this circumstance is fundamental in this sense that although the interpretation function maps expressions of **L** into their extensional values, it does so relatively to intensions. This fact explains Tarski's view (see above) that expressions of formalized languages always have concrete and intelligible (for us) meanings. The reason is that formalization does not depart from meanings possessed by expression before its undertaking and finishing.[8] The role of pragmatics is basic in this respect—in fact, metalinguistic explications of meaning typically follow usages of words and their complexes by people, particularly by competent users of language.

One can ask for a need of model-theoretic semantics in the situation that it invokes intensions. The answer points out that the construction of a model only on the intensional base appears as insufficient. It was confirmed by the discovery of non-standard models of arithmetic of natural numbers. This was done by considering model-theoretic constructions in which non-standard natural numbers greater than all numbers from the sequence 0, 1, 2, 3, ..., n, ... exist. The theory of the non-standard model has the same axioms as the usual arithmetic. Accordingly, semantic (semiotic) analysis of Peano axioms was not sufficient for the discovery of the non-standard model. On the other hand, the meaning of the term "natural number" was essential in this case as well. If a language is richer, the role of intensions become greater—the passing from propositional logic to predicate calculus, and further, e.g. the arithmetic of integers provides a good illustration here. To sum up, we need to consider that the meaning of **L** is the next limitation of formal semantics, because it must be done in an informal metalanguage. **ML** is of course normalized

[8] One should remember that Tarski was not entirely univocal in this respect. On one hand, he made various remarks on meanings, but, on the other hand, he expressed critical opinions about this notion and recommended its omission in semantics (Kokoszyńska, 1936, Tarski, 1936).

relatively to some explicit tasks, for instance, mathematical ones, but it is still not formalized, well at least partially. Gilbert Ryle (1953) also pointed out that words belonging to specialized terminologies (languages) have their ordinary meaning—in this case, "ordinary" means the same as "standard". This category is openly pragmatic. Ryle's observation justifies, to some extent, my earlier observation that pragmatics precedes semantics.

So-called limitative theorems (see Murawski, 1999), in particular, Gödel's first incompleteness theorem and Tarski's theorem on undefinability of truth, are still another manifestation of limitations of formal semantics. Consider formalized arithmetic of natural numbers (**AR**). If it is consistent, it is incomplete, that is, the sentences A and $\neg A$, expressed in its language, such that both are not provable in **AR**. Let the symbol **Tr** denote the truth-predicate for **AR**. In other words, the set of truths of **AR** is the extension of **AR**. Tarski's theorem says that **Tr** is not definable in **AR**. Both these results imply together that the semantic concept of truth cannot be reduced to the syntactic notion of proof. Another manifestation of this situation is the version of Gödel's theorem stating that the set of arithmetical truths is not finitely axiomatized—the concept of axiomatizability is also syntactic. The situation is actually intriguing, because the syntax of $\mathbf{ML}^{\mathbf{AR}}$ (the metalanguage of **AR)** can be arithmetized, that is, reduce to $\mathbf{L}^{\mathbf{AR}}$. One can see the reduction to arithmetic syntax (I do not consider the general case), means the full formalization of what is reduced. Now we immediately see that the full formalization of semantics is impossible.

Observations from the last paragraph throw some light onto the meaning of the adjective "formal" in the expression "formal semantics". If we say that the model theory of **AR** functions as a formal theory of this theory, it means only that **AR**-semantics employs various devices, for instance, set-theoretical or algebraic, which are located in the metalanguage which is always less formalized than the language of the object-theory. In the case considered, semantic $\mathbf{ML}^{\mathbf{AR}}$ is less formal than the syntactic $\mathbf{ML}^{\mathbf{AR}}$—the latter is subjected to arithmetization. Additionally, whereas Gödel's first theorem has a semantic (constructive and finitary even in a very restricted sense) proof, Tarski's theorem has to employ infinitary devices and thereby is non-constructive. Accordingly, semantics transcends (in the sense customary in philosophy) the syntax and appears as a fundamental fact, closely related to the situation that the former contains more informal ingredients than the latter. Although one could say that the syntax is limited as compared with semantics due to its (syn-

tax) conceptual simplicity smaller that semantic, but it does not change the opinion that limitations of formal semantics, constitute an intriguing phenomenon just pictured by its formality. A philosopher could say that it is a sign of the priority of content over form, but it does not mean that syntax is a complete arbitrary game. Formal semantics suggests that both, content and form, should be normalized by formalization. And this is a virtue for formalizing languages and theories.

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