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SYNTAX–SEMANTICS INTERACTION IN MATHEMATICS

SUMMARY: Mathematical tools of category theory are employed to study the syntax-semantic problem in the philosophy of mathematics. Every category has its internal logic, and if this logic is sufficiently rich, a given category provides semantics for a certain formal theory and, vice versa, for each (suitably defined) formal theory one can construct a category, providing a semantics for it. There exists a pair of adjoint functors, Lang and Syn, between a category (belonging to a certain class of categories) and a category of theories. These functors describe, in a formal way, mutual dependencies between the syntactical structure of a formal theory and the internal logic of its semantics. Bell's program to regard the world of topoi as the *univers de discours* of mathematics and as a tool of its local interpretation, is extended to a collection of categories and all functors between them, called "categorical field". This informal idea serves to study the interaction between syntax and semantics of mathematical theories, in an analogy to functors Lang and Syn. With the help of these concepts, the role of Gödel-like limitations in the categorical field is briefly discussed. Some suggestions are made concerning the syntax-semantic interaction as far as physical theories are concerned.

KEYWORDS: philosophy of mathematics, categorical logic, syntax-semantic interaction, Bell's program, Gödel-like limitations.

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