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Rozprawa

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ALL THE SUPERHERO'S NAMES

Summary: In this paper I concern myself with *The Superman Puzzle* (the phenomenon of the substitution failure of co-referential proper names in simple sentences). I argue that the descriptive content associated with proper names, besides determining the proper name's reference, function as truth-conditionally relevant adjuncts which can be used to express a manner, reason, goal, time or purpose of action. In that way a sentence with a proper name "NN is doing something" could be understood as "NN is doing something as NN" (which means "as-so-and-so"). I argue that the substitution of names can fail on modified readings because the different descriptive content of proper names modifies the main predicate differently. Here I present a formal representation of modified predicates which allows one to model intuitively the different truth-conditions of sentences from *The Puzzle*.
Keywords: The Superman Puzzle, proper names, substitution failure, qualifying prepositional phrases, modified predicates, descriptivism, adjuncts, pseudonyms, simple sentences

1. INTRODUCTION: DOUBLE LIFE

By the 1970s, Romain Gary, the French novelist, was a literary celebrity. A decorated war pilot and diplomat he won the Prix Goncourt in 1956, at the beginning of his career as a novelist. But

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twenty years later, critics and readers were sated with the books of a fading literary star. So, while still publishing as Romain Gary, he created a new identity, that of a young Algerian student, Émile Ajar, who had fled to Brazil to escape jail and from where he was sending his manuscripts. In 1975, the second of Ajar's novels became a literary sensation and the Académie Goncourt awarded the prize to the author whilst knowing nothing about his real identity. In such a way Gary became the only person to win the Prix Goncourt twice. Knowing that Gary and Ajar is one and the same person, consider:

(1) Romain Gary won the Prix Goncourt in 1956.

(1′) Émile Ajar won the Prix Goncourt in 1956.

(2) Émile Ajar, not Romain Gary, won the Prix Goncourt in 1975.

(2′) Romain Gary, not Émile Ajar, won the Prix Goncourt in 1975.

Sentences (1) and (2) are true but our intuitions about the truth-value of (1′) and (2′) are mixed. On the one hand, Gary and Ajar is one and the same person and it is true about this person that he won the prize in 1956 and in 1975, but on the other hand, while being Romain Gary, he didn't win the prize as Romain Gary in 1975, and didn't win it as Ajar in 1956.

Here is another story. The greatest boxer Muhammad Ali lost five fights in his boxer-career but he never lost a fight before he changed his name. Consider:

(3) Cassius Clay was never beaten, whereas Muhammad Ali lost five times.

(4) Muhammad Ali lost more fights than Cassius Clay.

Sentence (3) and (4) could be true (actually that is how people complain on Ali's fanpages) but again you may have mixed intuitions about their truth-value. Sentences (1)–(4) exemplify three main cases of *The Superman Puzzle* – the phenomenon of the substitution failure in simple sentences which occurs when a change from one co-referential name to another affects the truth-value of a sentence in an extensional context. By Case 1 (C1) I will understand a situation in which one and the same person (or object) with names "NN" and "MM" simultaneously does something as NN and does something else as MM (or does something as NN but does not act as MM (while still being MM)). C1 is represented by sentence (2) – the same person, Romain Gary, won the Prix as Émile Ajar, not as Romain Gary, while still being

Romain Gary. By Case 2 (C2) I will understand a situation in which the same person (or object) does something as NN at one time and does something as MM at another time (sentence (3)). Finally, all sentences with comparative quantifiers (e. g. "more than" in sentence (4)) will constitute Case 3 (C3).

I have presented three cases of *The Puzzle* using genuine proper names, not pseudonyms, but most examples you can find in the philosophical literature concern the names of superheroes:

C1:

"While talking on the phone to Superman, Lois looked through the window at Clark Kent" (Moore 1999, p. 102).

"Clark Kent went into phone booth and Superman came out" (Saul 1997, p. 102).

C2:

"I never made it to Leningrad, but I visited St Petersburg last week" (Saul 1997, p. 103).

C3:

"Superman was more successful with women than Clark Kent" (Saul 1997, p. 103).

"Superman leaps tall buildings more frequently than Clark Kent" (Moore 1999, p. 92 n. 1).

"Hammurabi saw Hesperus more often than he saw Phosphorus" (Crimmins 1998, p. 19).

Note that *The Puzzle* appears only for those who know that names "NN" and "MM" refer to one and the same person (Moore (1999) proposed calling such people "enlightened"). So if you are enlightened it seems that you have to choose between two ways of explaining why intuitively the substitution of co-referential proper names fails in sentences (1)–(4) and why sentences as (2), (3) and (4) seems true. You can say that the truth-values of (1) and (1′) differ because these sentences express different propositions (that is exactly why (2), (3) and (4) are true – they express a proposition other than an analytically false one). Or, on the contrary, you can say that sentences (1) and (1′)

semantically expresses one and the same proposition but pragmatically convey different ones, and that is why people have mixed intuitions about the truth-conditions of sentences (1)–(4). I will call a view of the former type semantic and of the latter type pragmatic.

The plan of this paper is as follows. In the next section I shall explain why *The Puzzle* puzzles. In section 3 I briefly explain my proposal of semantics for qualifying prepositional "as"-phrases ("as so-and-so") which I will analyze in a similar way as adverbs are treated – as predicate modifiers. In section 4 I lay the groundwork for my own proposal. I will develop a hypothesis that the descriptive content of proper names could behave as truth-conditionally relevant adjuncts and be an additional contribution of proper names to the truth-conditions. Finally, in the Appendix, I will present a formal semantics for predicate modifiers and a model for one of *The Puzzle* sentences.

2. WHY *THE PUZZLE* PUZZLES

Let me start from the semantic type of view. The proponents of such a view assume that sentences from *The Puzzle* express different propositions so the core of the puzzle lies in giving a semantic explanation as to why sentences which seem simple, differing in co-referring proper names only, nevertheless express different propositions. Let us have a closer look at such a sentence. Consider: "Superman is successful with women but Clark Kent is not". It seems at first glance that if you accept the Leibniz Law of the Indiscernibility of Identicals you face a dilemma: either you have to give up names' co-referentiality, or have to accept the view that such sentences are always false. Link – who was trying to solve the similar puzzle of substitution failure between co-referential group terms and between coextensive plural terms – expresses the former possibility in the following way (1983, p. 304): "So if we have, for instance, two expressions *a* and *b* that refer to entities occupying the same place at the same time but have different sets of predicates applying to them, then the entities referred to are simply not the same". If you give up the co-referentiality of names then the problem of substitutivity failure becomes trivial. In the case of *The Superman Puzzle*, David Pitt (2001), Bjørn Jespersen (2006) and (a contextual version of it) Joseph Moore (1999) hold such a view. According to them, sentences from *The Puzzle* express different propositions because proper names are not genuinely co-refering (they refer to different fusions of time-slices (Pitt) or to different aspects (Moore) of the same individual, or they refer to different individual concepts (Jespersen)). It is little wonder that giving up co-referentiality leads to problems with identity statements. Identity statements expressed by sentences of the form "NN is MM" come out false (or at least are false in some contexts). Besides this unintuitive consequence, this type of a solution blocks the substitution of proper names in situations in which it is intuitively allowed (Predelli 2004, p. 110; Saul 2000, p. 256; Saul 2007, pp. 33–34).

So perhaps it would be better to keep the co-referentiality of names and, in order to explain how sentences from *The Puzzle* could express different propositions, to give up the claim that sentences are simple (to give up the principle called by Predelli (2004, p. 108) "Syntactic Innocence"). Such a line of explanation was taken by Graeme Forbes (1997, 1999, 2006) who noticed that sentences from *The Puzzle*, as for example, "Lex fears Superman", could be paraphrased with the pronoun "such", "Lex fears Superman as such" (2006, pp. 157–58). According to Forbes, in the case of substitution failure, simple sentences should be understood as containing the covert prepositional phrase "as such" in which the pronoun "such" should be treated as a case of logophora (a special case of anaphora in which an expression serving as antecedent is taken itself as a referent of an anaphoric pronoun). In a nutshell, the Forbesean idea was to treat dossiers of information (or, more precisely, a capacity to activate a certain dossier) as a representation of Fregean modes of presentation (2006, p. 158). A speaker could create different dossiers in which he stores different information about one and the same object. A proper name serves as a label for somebody's dossier; so if you substitute one proper name in a sentence for a different but co-referential one, you will change the reference of a covert pronoun while the referent of a name will remain the same. The new label will activate a different dossier so all you have to do to get a difference in truth-conditions is to connect expressions and dossiers (modes of presentation) with a special function which induces opacity and makes a mode of presentation which is connected with a name as part of the truth conditions (2006, pp. 158–59).

Mark Crimmins (1993, p. 273) raised an objection to the general version of this view (which covers belief ascriptions) and proposed the consideration of a story in which Lois encounters Superman in both guises but does not know either of his names. We can report for example: "Lois believes that Clark is in the building, but doesn't believe that Superman is in the building". Intuitively, this sentence is true, but the possibility of using Lois's unlabelled dossiers is ruled out on Forbes' account.

So perhaps a better idea would be to preserve both co-referentiality and syntactic simplicity and shift the criteria of evaluation. Stefano Predelli (2004) followed this line and noticed that sentences from *The Puzzle* could be uttered in different contexts with different focuses of conversation. It could be so that, due to a special focus of a conversation in a context, some contextually salient circumstances should be taken into account in order to decide if a proposition expressed by a sentence in this context is true or not. The use of a name in a context triggers some features of the name's bearer which are of importance due to the focus of a conversation. Taking these features into account, the conversation participants decide if a referent of a proper name belongs to the extension of a predicate or not. Note that we are talking about the features of one and the same referent of both names and, once these features are taken into account, nothing prevents the substitution of proper names (if all they contribute to truth-conditions is their referent). Saul (2007, pp. 55–56) objected that it is not clear what these circumstances are and how to use them in order to solve examples of C3.

Let us leave the semantic camp and see what the proponents of the pragmatic view would propose. According to such views, sentences from *The Puzzle* semantically express one and the same proposition but pragmatically convey different ones. Alex Barber (2000) tried to explain *The Puzzle* using Gricean notion of implicature. A speaker uttering "Superman is more successful with women than Clark Kent" semantically expresses an analytically false proposition but his conversational partner assumes that the speaker is preserving the Cooperative Principle and is talking as if he is one who is unaware that Superman is Clark Kent. Those who are unaware (unenlightened speakers) would, under foreseeable epistemic conditions (for example taking into account attributes of appearing), utter what the speaker uttered (2000, pp. 303–304).

But what about truth-conditions? As we know, an implicature is not a part of the truth-conditions of a proposition literary expressed. Consider:

(5) If Clark Kent didn't ever pick up a woman and Superman did, then Clark Kent is more successful with women then Superman.

We could have mixed intuitions about "Superman is more successful with women than Clark Kent" but sentence (5) strikes us as false (or even inconsistent). But it should be true on Barber's account (because it is an implication from false to false). So it seems that the pragmatic view leads to a dilemma: either the information pragmatically conveyed is a part of what is said and affects the truth-conditions or the truth-conditions of what is said differs radically from our intuitions. Note that if you accept the former claim (as Recanati (2012, p. 203 n. 5) did) you will owe the same explanation as the proponents of a semantic view.

So the main problem for a real pragmatist is to provide semantically adequate truth-conditions for sentences from *The Puzzle*. It has to be said that a lot of people have an intuition similar to Barber's in that enlightened speakers uttering such sentences somehow pretend. Thomas Zimmermann (2005) elaborated this intuition and tried to fix the problem with the right truth-conditions. In a nutshell, what makes speakers unenlightened is the lack of knowledge that NN and MM is one and the same person. So when enlightened speakers utter sentences from *The Puzzle* they pretend and talk as if they were unenlightened: "If I believed that NN is not MM then I would say that NN is Q". Zimmermann calls such utterances "counterfactual speech acts" (2005, pp. 77–78). According to him, in our conversational practice we naively assume that no two names of our language have the same bearer (*Principle of Uniqueness* (UP), 2005, p. 70). This assumption is rather a naive belief, nevertheless, according to Zimmermann, it is a cornerstone of our conversational behavior and constitutes one of the conversational principles. So when one enlightened speaker talks to another and uses two co-referential names, he violates one of the conversational principles and this in turn triggers an implicature that the speaker does so in order to convey another proposition. But what about truth-conditions? Let us recall Frege's criterion of thought difference (1892/1984, p. 162): "Anybody who did not know that the evening star is the morning star might hold the one thought to be true, the other false". According to this criterion, two sentences with co-referential names express two different thoughts (which are the same in terms of truth-value) and for somebody one of the thoughts could be true and the other could be false with respect to the things he believes. Zimmermann uses this criterion: sentences from *The Puzzle* have the same objective truth-value but could differ in truth-value with respect to somebody's doxastic perspective (differ in a subjective truth-value). So when an enlightened speaker violates UP he switches his language to the subjective language of unenlightened speakers who believe wrongly that "NN" and "MM" refer to different people. "Switching languages" is expressed formally as changing the context of uttering to another which is exactly the same except for the language it is spoken in. So we get intuitively right truth-conditions (sentence (5) appears false) in a subjective language of those who believes that names "NN" and "MM" refer to different people. This last claim makes this solution similar to the proposal of all of those from the semantic camp who assume that proper names do not genuinely co-refer and that is why they have a similar problem with the falsity of identity statements (2005, pp. 94–95).

I hope I have convinced you that *The Puzzle* puzzles and now I intend to present my solution to it.

3. MODIFIED PREDICATES

I will remain in the semantic camp and develop an idea similar to the Forbesean. I take *The Superman Puzzle* to be a case of a broader phenomenon of substitution failure of co-referential nominal phrases: apart from proper names, this phenomenon concerns co-referential group terms (*The Committee Puzzle*), plural terms, definite descriptions and natural kind terms (Link 1983; Landman 1989; Szabó 2003). In (Poller 2016) I raised a hypothesis that the role of a descriptive content associated with proper names (and other terms) could not only be reference determining but this content could also serve as a truth-conditionally relevant adjunct used to express a manner, reason, goal, time or purpose of action. The idea in a nutshell is to treat identifying descriptions "the so-and-so" associated by speakers with a proper name as qualifying prepositional phrases "as so-and-so". In such a way, a sentence containing a proper name "NN is doing something" could be understood as "NN is doing something as NN" (which means as so-and-so). I present the semantics of prepositional "as"-phrases briefly (elaborated version of it you can find in (Poller 2016)) and then turn to a way of how it could be used to solve *The Puzzle*.

Consider the following sentence:

(6) The papal nuncio supported an anarchist protest.

Although nothing prevents one understanding (6) as saying the papal nuncio supported an anarchist protest as a private person you understand (6) rather as (6′):

(6′) The papal nuncio supported an anarchist protest as the papal nuncio.

We could paraphrase (6′) as (6′′):

(6′′) The papal nuncio as such supported an anarchist protest.

I agree with Forbes who noticed that all the sentences which form *The Puzzle* could be paraphrased with the pronoun "such" (cf. "Lex fears Superman as such", 2006, p. 158) and espouse the view that "as"-phrase invokes a mode of presentation connected with an expression. But contrary to Forbes, who treats "such" as a case of logophora, I think of "such" as an adjectivally anaphoric pronoun standing for a property (after (Carlson 1980), (Landman & Morzycki 2003), (Landman 2006), (Siegel 1994), (Wood 2002)) and see no reason to think that the preposition as induces opacity. I propose analyzing prepositional "as"-phrases in a similar way to that in which adverbs are analyzed – as predicate modifiers[1].

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[1] An anonymous referee noted that a placing syntactically "as"-phrase as a predicate modifier (John as a miner supported a protest) seems unintuitive, and the "as"-phrase should be analyzed as a name-modifier instead (John as a miner supported a protest). Such a line of analysis was used by Landman (1989). Szabó (2003, p. 391) raised convincing syntactical objections against such a view: modified names ("John-as-a-miner") do not coordinate with other names, cannot form possessives and cannot be given as an answer for "who"-

In my analysis of "as"-phrases, I followed Romain Clark (1970) who proposed a semantics for adverbs and prepositional phrases which was an alternative to events semantics proposed by Donald Davidson (1967/2001). The core of Clark's proposal is the idea that predicates could be built recursively out of n-place predicate constants by adding modifiers which have i places in total. So for example take "stroll". It is a 1-place predicate. Take the adverb "slowly". If you add this adverb to "stroll" (getting "slowly stroll") you would not increase the number of argument places. So "slowly" is 0-place modifier (as are many other adverbs). The extension of "slowly stroll" is a subset of the extension of "stroll" (Clark 1970, p. 325) and that is why you can infer from "Sebastian slowly strolled" that "Sebastian strolled" but not the other way around. This type of adverbial entailment failure is known as *Non-Entailment* (Davidson 1967/2001; Katz 2008) and we will see that it is a key property in solving the failure of the substitution puzzle. Now take "at" and "through". Each of them are 1-place modifiers and if you add them to "stroll" (getting "stroll-through-at") you will increase the number of argument-places and will get a new 3-place predicate out of a 1-place initial one. You can infer from "Sebastian strolled through the streets of Bologna at 2 a.m." (Davidson 1967/2001, p. 167) that "Sebastian strolled" because the new 3-place predicate is connected with the initial 1-place predicate "stroll" by a requirement that an object occupying the first place of the triple (Sebastian) should belong to the extension of "stroll" (this type of entailment is called *Drop*).

I propose treating prepositional "as"-phrases as 0-place predicate modifiers. Unlike other prepositional phrases, "as"-phrases do not increase the number of argument-places, and, unlike adverbs, do not modify a predicate with all its argument places a s a w h o l e, they modify it on one argument-place only. Note that if you know that d is doing A and B and *is* φ, you can't infer that either A or B is done by d as φ (by *Non-Entailment*). This entailment failure shows that the extension of a modified predicate *doing* A *as* φ although dependent on the extensions of A and φ (by *Drop*), is not fully determined by them.

questions. Taking these arguments into account I analyze "as"-phrases as predicate modifiers. I answered syntactic and semantic objections raised by Szabó against such a view in (Poller 2016).

Let me briefly go through some syntactic and semantic definitions. By a modifier we will understand all predicates abstracted[2] from an atomic formula or a conjunction of atomic formulas with one free variable, e.g. $\lambda x.Q(x)$, $\lambda x.\big(P(x) \wedge Q(x)\big)$. An n-place predicate constant Q could be modified by a modifier $(\lambda x.\varphi)$ on its ith argument place; we write this new modified predicate as "$Q^i_{\lambda x.\varphi}$". For example "greet" is a two-place predicate, $\varphi(x)$ is a formula with one free variable in which φ means "a host of a party". $greet^1_{\lambda x.\varphi}$, $greet^2_{\lambda x.\varphi}$ are predicates built via modification from the predicate constant $greet$; we read them "as a host of a party x greets y" (modification on the 1st argument place) and as "x greets y as a host of a party" (modification on the 2nd argument place). We will use a simplifying convention and in the case that a modifier is a predicate abstracted from an atomic formula, $P(x)$, we will simply write "Q^i_P" instead of "$Q^i_{\lambda x.P(x)}$" and in the case that Q is 1-place predicate we will write "Q_P" instead of "Q^1_P".

I limit predicates abstracts which could be modified to predicates abstracted from atomic formulas and their negations, $\lambda x.Q(z_1, \dots, z_n)$ and $\lambda x.{\sim}Q(z_1, \dots, z_n)$. A modifier $\lambda y.\psi$ modifies a predicate abstract on ith argument place of Q (written "$(\lambda x.\varphi)^i_{\lambda y.\psi}$" in general notation). I preserve an intuition that a modified predicate abstract $\big(\lambda x.Q(z_1, \dots, z_n)\big)^i_{\lambda y.\psi}$ and a predicate abstracted from a formula with a modified predicate $\Big(\lambda x.Q^i_{\lambda y.\psi}(z_1, \dots, z_n)\Big)$ are one and the same predicate (so you can take a modifier "in and out" of a predicate abstract, see (Poller 2016) for proof). Formulas with all kinds of predicates (predicate constants, predicate abstracts, modified predicates and modified predicate abstracts) are built in a standard way.

Let Q and P be 1-place predicates. I defined an interpretation of modified predicate Q_P ("$Q\ as\ P$") as a subset of a conjunction of interpretations Q and P: $I(Q\_P) \subseteq (I(Q) \cap I(P))$. So, for example, d could be the papal nuncio $\big(d \in I(P)\big)$ and could support an anarchist protest $\big(d \in I(Q)\big)$, but could support an anarchist protest not as the papal nuncio $(d \notin I(Q_P))$. (For the general definition of an interpretation of a modified predicate see *Def.* III.*S*7 in Appendix). A modified predicate is still a predicate, it is interpreted as a subset of a predicate being modified, that is, a set of n-tuples such that every ith

[2] In using "predicates abstracted from a formula", "predicate abstracts" I followed Fitting and Mendelsohn (1998, pp. 194, 196 Definition 9.4.2).

element in n-tuple fulfils the descriptive content φ. Modifiers are closed under the conjunction: $I(Q^{i}_{\lambda x.(\varphi\wedge\psi)}) = I(Q^{i}_{\lambda x.\varphi}) \cap I(Q^{i}_{\lambda x.\psi})$.

My analysis covers uses of "as"-phrases as adjuncts of manner ("I will use the rest of the olive oil as a base for salad dressing"), time ("Ann was fat as a child"), reason ("As a firefighter, John was asked to help in the rescue action") and purpose ("They hired him as a launching engineer"). But it doesn't cover uses of "as"-phrases as adjuncts of comparison ("His mother still treats him as a child") when we compare two things A and B under respect C and do not say that A is B (contrary to requirements of our semantic definition).

4. NAMES AND PSEUDONYMS AS MODIFIERS

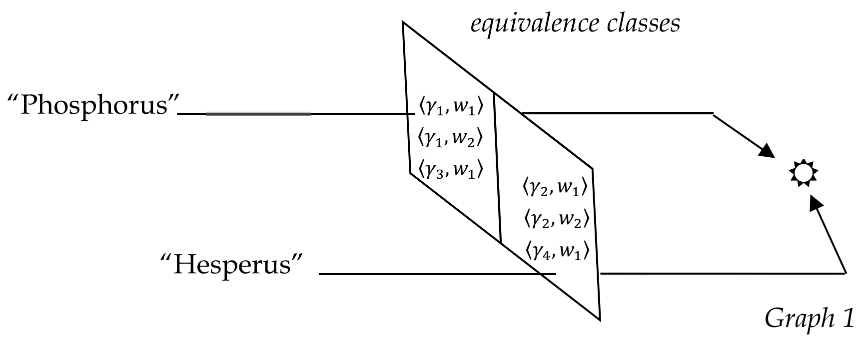
Let us return to sentence (1′), "Émile Ajar won the Prix Goncourt in 1956". The reason why we may have mixed intuitions about its truth conditions lies in the ambiguity between modified and unmodified readings. You can say, "It's true that Ajar won the Prix Goncourt in 1956, but Ajar won the Prix not as such but as Romain Gary". The possibility of replacing a proper name with the adjectivally anaphoric pronoun "such" supports the claim that a proper name in an "as"-phrase ("didn't win as Émile Ajar") is understood as standing for a property, so the predicate "win" is modified not by a proper name but by the descriptive content of a proper name. The idea standing behind the modification of predicates by names is simple: the modifying content of a proper name n is a predicate $\lambda x.\varphi$ abstracted from the formula φ of a definite description $\iota y.\varphi$ connected with a proper name n.

Despite being a descriptivist (in my opinion, speakers do associate definite descriptions with proper names) I do not think that the phenomenon of predicate modification by a descriptive content of names should be understood as evidence supporting descriptivism. Possibly you can accept this phenomenon without accepting any version of descriptivism (however, you will need an additional explanation of what kind of descriptive content should be semantically connected with names and why). Because of my claim that the modifying content of a proper name is a property expressed by a description connected with the name, I need to briefly explain my proposal of the formal representation of proper names in accordance with the descriptive theory of reference (descriptions are used to fix a name's reference, a full version of this proposal can be found in (Poller 2014)). In a nutshell I represent proper names formally as a special kind of terms (which I call "name-terms") which designate via sets of definite descriptions. By "definite description" I understand a special kind of iota-terms of the form $\iota x.[_i]\varphi$, where "$[_i]$" is a notational variant of $\mathbf{then}_i$ operator ("true at t_i") taken after (Rini & Cresswell 2012). Time operator $[_i]$ fixes a time of evaluation, so a definite description $\iota x.[_i]\varphi$ designates with respect to any time t the object designated by iota-term $\iota x.\varphi$ with respect to time t_i (I call definite descriptions $\iota x.[_i]\varphi$ actual with respect to t_i). I am trying to catch the idea that a definite description designates contingently with respect to possible worlds but if it designates in a world, it designates in that world one and the same object with respect to any time. That is why a iota-term representing a definite description should have a fixed time-parameter (e.g. "the present Pope", "the Pope in 1967").

My account of modified predicates is not general so the most complicated modifier could be a predicate abstracted out of a conjunction of atomic formulas with one free variable. That is why I will use only some of the iota-terms $\iota x.[_i]\varphi$, such that φ is a conjunction of atomic formulas. To avoid circularity (to be sure that definite descriptions $\iota x.[_i]\varphi$ used to determine a name-term's reference contain no name-terms) I need two languages, $\mathcal{L}$ and $\mathcal{L}^+$ ($\mathcal{L} \subset \mathcal{L}^+$). Let me start from language $\mathcal{L}$ which contains only variables and iota-terms as terms. The idea is to let name-terms designate through equivalence classes of descriptions designating one and the same object. But descriptions designate different objects with respect to different worlds so we need to define an equivalence relation not on a set of descriptions but on a set of pairs containing a description and a world in which the description designates. In order to be able to formally distinguish two co-referential names I have added a set of predicates (N_1, N_2, N_3, ...) to $\mathcal{L}$ which we will read as "called α", "called β" etc. where "α", "β" are string of sounds or inscriptions.[3] I will use symbol "$!x.\varphi$" for iota-terms $\iota x.\varphi$ with only one variable x which occurs free in φ. Letting the

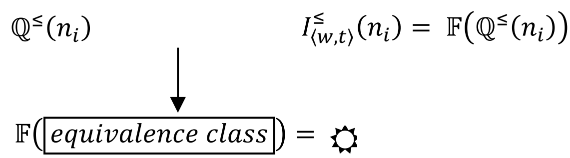
[3] Arguments supporting such a view of verbs of naming can be found in (Geurts 1997), see also (Matushansky 2008).

formula φ in a description $!x.[_i]\varphi$ have a form of a conjunction of a distinguished predicate and a 1-place undistinguished predicate ($\varphi = (N_i(x) \land Q(x))$, e.g. "a planet called [fɒs fər əs]") we can define an equivalence relation in such a way that two description-world pairs belong to the same class when their descriptions designate the same object and contain the same predicate N_i. So for example, take two descriptions, "the planet called [fɒs fər əs]", "the planet called [hɛs pər əs]" (we name them γ_1, γ_2 respectively). Both descriptions γ_1, γ_2 designate in our world w, but pairs $\langle\gamma_1, w\rangle$, $\langle\gamma_2, w\rangle$ will belong to different equivalence classes because γ_1 contains predicate "called [fɒs fər əs]" while γ_2 contain a different predicate "called [hɛs pər əs]". This idea is represented schematically in *Graph 1* below:



Graph 1

I won't go into formal details (full versions of theses definitions can be found in Appendix) and instead will just explain the key steps. In order to define an interpretation of a name-term n_i I need two functions – one which connects n_i with an equivalence class (function $\mathbb{Q}^{\leq}$) and the other which takes an equivalence class and gives the object designated by every description in the class (function $\mathbb{F}$). I have presented this idea in *Graph 2* below:



Graph 2

In effect, name-terms designate rigidly (see (Poller 2014) for proof) and are not synonymous with descriptions (this is exactly what a descriptive theory of reference postulates). As I have said, the idea of the predicate modification by a descriptive content of a proper name is simple: if we say that NN is doing something as NN, we mean that there is a (unspecified) way of describing NN such that NN is doing something in that way. Let φ stand for an atomic formula or a negation of an atomic formula. We will take any name-term n to be a modifier, and write "$(\lambda x.\varphi)^i_n$" for a modified predicate abstract and "$(\lambda x.\varphi)^i_n(n)$" for a formula (a name-term and a predicate abstract modified by a name-term could form a formula iff the name-term occupying an argument place of this predicate is the same as the modifying name-term). The idea of predicate modification by a descriptive content of a proper name is represented formally as a requirement that a formula $(\lambda x.\varphi)^i_n(n)$ is satisfied in a model with respect to a world w and a time t_j iff there is a description $!y.[_j]\psi$ in the set of descriptions for the term n and the world w such that the model satisfies $(\lambda x.\varphi)^i_{\lambda y.\psi}(n)$ with respect to $\langle w, t_j \rangle$. Note that we drop a fixing-time operator $[_j]$, so our modifying descriptive content ($\lambda y.\psi$ obtained from a definite description $!y.[_j]\psi$) is sensitive to scope differences of temporal and modal operators. Take a formula with a name and a predicate. On an unmodified reading, $(\lambda x.\varphi)(n)$, all that the descriptive content of a proper name does is just pick up the reference, that is why a change of a proper name to a different but co-referential one is without significance because all you need for truth-conditions is just the name's referent and a property named by a predicate. But on a modified reading, $(\lambda x.\varphi)^i_n(n)$, we want the descriptive content of a name to be taken into account as a circumstance of action (expressing a manner, goal, reason or time), so we make it a part of a predicate. When we say that NN is doing something as NN we understand by it that NN is doing something in a descriptive way ψ actual with respect to a time (and a world) of evaluation. So, for example, by saying (3), "Cassius Clay was never beaten, whereas Muhammad Ali lost five fights", we convey that the greatest boxer was never beaten at a period of time when he was a boxer called "Cassius Clay" and he lost five fights after changing his name to "Muhammad Ali".

It has to be said that on this account a descriptive content of co-referring genuine proper names differs only in naming predicates ("called α", "called β"). Intuitively the difference in descriptive content between "Superman" and "Clark Kent" is deeper. I take expressions such as "Superman" or "Batman" to be pseudonyms and think that the semantics of proper names differs from the semantics of pseudonyms (cf. Katz 2001). Let us have a closer look at pseudonyms. They are broadly understood as the names that people assume for a particular purpose (Room 2010, p. 3). In American copyright law it is underlined that a pseudonym should be fictitious (nicknames and other diminutive forms of legal names are not considered as fictitious, cf. Copyright Office Fact sheet FL101). Usually people take pseudonyms for their activity as artists, writers, political and religious leaders, gamers, secret agents and so on. It is a remarkable fact about pseudonyms that they can become an adopted new name whenever a person becomes mainly or solely known by their pseudonym (Room 2010, p. 4). I take this feature of pseudonyms – to be assumed for a particular purpose – as a key feature that distinguishes pseudonyms from genuine names.

As I explained earlier, I represent genuine proper names as name-terms which designate via sets of definite descriptions of the form $!x.[_i](N(x) \wedge Q(x))$. The key difference between the formal representation of pseudonyms and names lies in representing pseudonyms as terms (called "pseudonym-terms") which designate via sets of definite descriptions of the form $!x.[_i](N_P(x) \wedge Q_P(x))$. Every description in such a set contains a modified distinguished predicate N_P, which we read as "named α as P" (e.g. "called [benɪdɪkt] as a pope", "called [ɹɑkɪt] as a hockey player"), and contains a 1-place undistinguished predicate modified by the same predicate P. By such a formal representation of pseudonyms I am trying to express their key feature of being assumed for a particular purpose. So I want the descriptive content of a pseudonym to describe an individual as doing everything with this particular purpose (e.g. "called [benɪdɪkt] a s a p o p e", "sends a message to the faithful a s a p o p e", "publishes a work a s a p o p e" etc.). The other key feature of pseudonyms, their possibility of becoming genuine names (e.g. "John Wayne"), when a person starts to use a pseudonym not only for a particular purpose, will not be formally represented.[4, 5]

As I said earlier, to avoid circularity I need two languages, $\mathcal{L}$ and $\mathcal{L}^+$ ($\mathcal{L} \subset \mathcal{L}^+$). Language $\mathcal{L}$ contains only variables and iota-terms as terms and language $\mathcal{L}^+$ contains additionally a set of name-terms $\mathcal{N} = \{n_1, n_2, n_3, \dots\}$ and a set of pseudonym-terms $\mathcal{M} = \{m_1, m_2, m_3, \dots\}$. Pseudonym-terms are interpreted in the same way as name-terms – via

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[4] However, the possibility of pseudonyms to become genuine names could be formally represented. In order to represent it we could add a special operator "only" ($*$) operating on a modifier "only as P"). For example, at the beginning of his actor career Marion Morrison was named [dʒɒnwān] only as a film actor but from a time t_i he was named [dʒɒnwān] not only as an actor. So if we let pseudonym-terms designate via sets of definite descriptions of the form $!\,x.\,[_j](N_{*P}(x) \wedge Q_P(x))$ (containing a distinguished predicate modified by the "only as P" modifier, N_{*P}), then from the time t_i it would be false that Morrison is named [dʒɒnwān] only as a film actor. A pseudonym-term (formal representation of "John Wayne") is obstinately rigid and designates Morrison with respect to any time and world but from the time t_i (in our world w) it has no descriptive content which could modify a predicate (since t_i it is false that he is named [dʒɒnwān] only as an actor which in turn means that there is no description of the form $!\,x.\,[_k](N_{*P}(x) \wedge Q_P(x))$, where $i \leq k$, connected with the pseudonym-term). Letting name-terms designate via descriptions containing modified predicates we will get a name-term formally representing the name "John Wayne" (not the pseudonym "John Wayne") which would designate via descriptions with fixing-time operators $[_k]$, where $i \leq k$. This means that at any time later than t_i Morrison would not do anything under the pseudonym but under the name "John Wayne".

[5] I need to note that things are not so simple from the formal side. Imagine that Smith decided to be named "Rocky" as a boxer. Intuitively, besides the pseudonym "Rocky", he did not take a new name "Rocky". Formally we will have descriptions designating Smith with "named [ɹɑkɪ] as a boxer"-predicate and with unmodified "named [ɹɑkɪ]"-predicate. Due to this besides a pseudonym-term designating Smith we will have a name-term designating him via descriptions containing "named [ɹɑkɪ]"-predicate. In effect we will have name-terms which do not model any proper names from a natural language. In order to prevent such consequences we need to "throw away" intuitively "rubbish" descriptions containing the unmodified predicate "named [ɹɑkɪ]" and designating Smith (see *Def.* VI.$S(c)$, $S(d)$ and Δ^*). I have elaborated upon the problem of "rubbish" descriptions in my PhD thesis (2014).

equivalence classes of description-world pairs, $I^{\leq}_{\langle w,t \rangle}(m_i) = \mathbb{F}(\mathbb{Q}^{\leq}(m_i))$, which means that pseudonym-terms are obstinately rigid. A formula with a pseudonym-term is satisfied in a standard way when the referent of a pseudonym belongs to the extension of a predicate. However, a pseudonym-term has a specific feature which distinguishes it from a name-term: in all possible worlds such that a set of descriptions determining the pseudonym's reference is non-empty a pseudonym-term's referent would have a property "P" besides a property "called α". Let me illustrate this specific feature by the following example. Consider four possible worlds w_1, w_2, w_3, w_4. In world w_1 Joseph Ratzinger became pope and as pope was called [benıdıkt sıksti:nθ]. On becoming pope, he visited Germany first. In world w_2 he, Benedict XVI, visited France first. In world w_3 Ratzinger failed to get into theological school and became a cigarette smuggler who always left sixteen cigarettes in his abandoned caches and as a result was known in the criminal underworld as Benedict 16. In world w_3 the police were unable to catch him but in world w_4 Ratzinger, called [benıdıkt sıksti:nθ] as a smuggler, was arrested. Formally we will have two pseudonym-terms representing Benedict XVI-a pope and Benedict-16-a smuggler pseudonyms. In all worlds such that Ratzinger is called [benıdıkt sıksti:nθ] as a smuggler he is a smuggler. Contrary to pseudonyms, proper names have no specific property besides "called α" which is preserved in possible worlds in which a set of descriptions determining the name's reference is non-empty and that is why it is easier to construct *The Puzzle* using pseudonyms than proper names.

I defined predicate modification by a descriptive content of a proper name as a requirement that a formula $(\lambda x.\varphi)^i_n(n)$ is satisfied in a model with respect to a world w and a time t_j iff there is a description $!\,y.\,[_j]\psi$ in the set of descriptions for the term n and the world w such that the model satisfies $(\lambda x.\varphi)^i_{\lambda y.\psi}(n)$ with respect to $\langle w, t_j \rangle$. It seems that there is no reason for an intended definition of modification by a descriptive content of a pseudonym $(\lambda x.\varphi)^i_m(m)$ to be different. But, as we remember, the account of modified predicates presented here is not general and the most complicated modifier is a predicate abstracted from a conjunction of formulas containing unmodified atomic predicates. Every definite description connected with a pseudonym-term contains predicates modified by some predicate P,

$!\, x. [_i](N_P(x) \wedge Q_P(x))$, and predicate abstracted from it can't be used as a modifier. That is why a definition of predicate modification by a descriptive content of a pseudonym differs from a definition of a modification by a descriptive content of a proper name: a formula $(\lambda x. \varphi)^i_m(m)$ is satisfied in a model with respect to a world w and a time t_j iff there is a description $!\, y. [_j](N_P(x) \wedge Q_P(x))$ in the set of descriptions for the term m and the world w such that the model satisfies $(\lambda x. \varphi)^i_{\lambda y. P(y)}(m)$ with respect to $\langle w, t_j \rangle$. Having no modification of a predicate by an already modified predicate (having no iteration) we cannot, for example, express that Superman is entering the phone booth dressed *as a superhero* (predicate "entering" is modified by the adjunct "dressed" which in turn is modified by the "as"-phrase). Instead we express the fact that Superman is entering the phone booth *as a superhero* (predicate "entering" is modified by the "as"-phrase).

In the Appendix I have presented the formal semantics for modified predicates and have modeled sentences with names and pseudonyms representing C1. I have not presented a model for C2 sentences (sentences such as "I have never made it to Leningrad, but I visited St Petersburg last week") because they are easy to explain: intuitively such sentences are true because it is not the case that Petersburg is officially called [lenəngræd] anymore, so you can't visit it as such. Nor have I presented a model for C3 sentences with comparative quantifiers such as (4). Intuitively in (4) we compare the cardinality of sets of fights that the greatest boxer won as Muhammad Ali and won as Cassius Clay. The cardinality of these sets differs and that is why (4) is true.

CONCLUSION

I treat the phenomenon of the substitution failure of co-referential proper names in simple sentences as a special case of the broader phenomenon of a lack of substitutivity between two co-referential nominal phrases. I argue that the descriptive content associated with proper names, besides determining the proper name's reference, functions as truth-conditionally relevant adjuncts which could be used to express a manner, reason, goal, time or purpose of action. In that way a sentence with a proper name "NN is doing something" which could be understood as "NN is doing something as NN" (which means as-so-and-so). I propose to analyze qualifying "as"-phrases as predicate modifiers and present a formal representation of modified predicates. According to my view, sentences from *The Superman Puzzle* are ambiguous between modified and unmodified readings and this assumption explains why speakers have mixed intuitions about such examples. Whereas nothing prevents the substitution of co-referential proper names on unmodified readings, the substitution of names can fail on modified readings because the different descriptive content of proper names modifies the main predicate differently, so in effect sentences can have different truth conditions. I treat names such as "Superman" and "Batman" as pseudonyms and argue that the semantics for pseudonyms differs in some respect to the semantics for genuine proper names. Intuitively, the key difference between names and pseudonyms lies in a pseudonyms' feature of being assumed for a particular purpose and I reflect this feature in a formal representation of pseudonyms.

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APPENDIX: THE FORMAL REPRESENTATION OF NAMES, PSEUDONYMS AND MODIFIED PREDICATES

The languages $\mathcal{L}$ and $\mathcal{L}^+$ are based on first-order predicate logic with identity and descriptions (I followed Fitting & Mendelsohn 1998). I will skip all standard definitions and present the definitions that are specific for a formal representation of modified predicates, names and pseudonyms.

Let me start from the language $\mathcal{L}$ which contains only two sorts of terms, variables and iota-terms.

Definition I*: The alphabet of* $\mathcal{L}$
A first-order language $\mathcal{L}$ contains the following symbols: sentential connectives $\wedge$, $\vee$, $\rightarrow$, $\leftrightarrow$, $\sim$; quantifiers $\exists$, $\forall$; an infinite set of individual variables $x_1, x_2, x_3, \ldots$; an infinite set of predicate constants $P_1, P_2, P_3, \ldots$, with a positive integer (an arity) assigned to each of them; identity sign $=$; the definite descriptions operator ι; the abstraction operator λ; temporal operators of past **P** and future **F**; an infinite set of temporal operators $[_i]$ ("true at t_i"), where $i \in \mathbb{N}$; modal operators $\Box$, $\Diamond$; an infinite set of distinguished predicate constants $N_1, N_2, N_3, \ldots$; a set of numerical symbols for natural numbers; the left parenthesis (, the right parenthesis).

Definition II*: The syntax of* $\mathcal{L}$
Predicate constants and, defined below, predicate abstracts, modified atomic predicates and modified predicate abstracts are predicates of $\mathcal{L}$. An *atomic predicate* of $\mathcal{L}$ is any predicate constant. The notions of a formula, a term, a predicate and a free variable occurrence are defined as follows:

The notions of a variable ($R1$), a predicate constant ($R2$), an atomic formula ($R3$), $\sim\varphi$ ($R4$), $(\varphi \wedge \psi)$, $(\varphi \vee \psi)$, $(\varphi \rightarrow \psi)$, $(\varphi \leftrightarrow \psi)$ ($R5$), $\mathbf{P}\varphi$, $\mathbf{F}\varphi$, $[_i]\varphi$ ($R6$), $\Box\varphi$, $\Diamond\varphi$ ($R7$), $\forall_x\varphi$, $\exists_x\varphi$ ($R8$), $\iota x.\varphi$ ($R9$), $(\lambda x.\varphi)$ ($R10$) are defined in a standard way;

$R11.$ if Q is a 1-place predicate constant and x is a variable, then $(\lambda x. Q(x))$ is a modifier. Modifiers contain no free variable occurrences;

$R12.$ if $(\lambda x. \varphi)$, $(\lambda x. \psi)$ are modifiers, then $(\lambda x. (\varphi \wedge \psi))$ is a modifier;

$R13.$ if Q is a n-place predicate constant and $(\lambda x. \varphi)$ is a modifier then $Q^i_{\lambda x.\varphi}$ is n-place atomic predicate modified by $(\lambda x. \varphi)$ on ith argument place of Q (where $1 \leq i \leq n$);

$R14.$ if $(\lambda x. Q(z_1, \dots, z_n))$ is a predicate abstract and $(\lambda y. \psi)$ is a modifier, then $(\lambda x. Q(z_1, \dots, z_n))^i_{\lambda y.\psi}$ is a predicate abstract modified by $(\lambda y. \psi)$ on ith argument place of Q (where $1 \leq i \leq n$); the free variable occurrences in $(\lambda x. Q(z_1, \dots, z_n))^i_{\lambda y.\psi}$ are those of $(\lambda x. Q(z_1, \dots, z_n))$;

$R15.$ if $(\lambda x. \sim Q(z_1, \dots, z_n))$ is a predicate abstract and $(\lambda y. \psi)$ is a modifier, then $(\lambda x. \sim Q(z_1, \dots, z_n))^i_{\lambda y.\psi}$ is a predicate abstract modified by $(\lambda y. \psi)$ on ith argument place of Q (where $1 \leq i \leq n$); the free variable occurrences in $(\lambda x. \sim Q(z_1, \dots, z_n))^i_{\lambda y.\psi}$ are those of $(\lambda x. Q(z_1, \dots, z_n))$;

$R16.$ if Q is a n-place predicate constant, $Q^i_{\lambda x.\varphi}$ is n-place modified predicate and $z_1, \dots, z_n$ is an n-element sequence of variables, then $Q^i_{\lambda x.\varphi}(z_1, \dots, z_n)$ is a formula in which all variable occurrences in the n-element sequence are free;

$R17.$ if $(\lambda x. \varphi)$ is a predicate abstract and s is a term, then $(\lambda x. \varphi)(s)$ is a formula; the free occurrences of variables in $(\lambda x. \varphi)(s)$ are those of $(\lambda x. \varphi)$ together with those of s;

$R18.$ if $(\lambda x. \varphi)^i_{\lambda y.\psi}$ is a modified predicate abstract and s is a term, then $(\lambda x. \varphi)^i_{\lambda y.\psi}(s)$ is a formula; the free occurrences of variables in $(\lambda x. \varphi)^i_{\lambda y.\psi}(s)$ are those of $(\lambda x. \varphi)^i_{\lambda y.\psi}$ together with those of s;

$R19.$ nothing else is a formula, a term, a predicate, a modifier and a free occurrence of a variable.

Notational convention:
– if Q is a 1-place predicate constant and β is a modifier, then instead of "Q^1_β" we will write "Q_β";
– if Q is a n-place predicate constant and $(\lambda x. P(x))$ is a modifier, then instead of "$Q^i_{\lambda x.P(x)}$" we will write "Q^i_P".

Definition III*: The semantics of* $\mathcal{L}$
A varying domain first-order model $\mathfrak{M}$ for $\mathcal{L}$ is a structure $\mathfrak{M} = \langle \mathcal{D}, T, <, W, I \rangle$, such that:
– $\mathcal{D}$ is a domain function mapping pairs of possible world and time $\langle w, t \rangle$ to non-empty sets. The domain of the model is the set $\cup \{\mathcal{D}_{\langle w,t \rangle}: w \in W, t \in T\}$. We write $\mathcal{D}_\mathfrak{M}$ for the domain of the model $\mathfrak{M}$ and $\mathcal{D}_{\langle w,t \rangle}$ for a value of the function $\mathcal{D}$ for an argument $\langle w, t \rangle$;
– T is a set of natural numbers and $<$ ("earlier then") is a linear order defined on elements of T (a set $(T, <)$ is thought as a flow of time);
– W is a non-empty set of possible worlds;
– I is a function which assigns an extension to each pair of an atomic predicate or modified atomic predicate of $\mathcal{L}$ and a pair $\langle w, t \rangle$, where $w \in W, t \in T$, in the following way:
– if Q is a n-place predicate constant, then $I_{\langle w,t \rangle}(P) \subseteq \mathcal{D}^n_\mathfrak{M}$;[6]
– $I_{\langle w,t \rangle}(=) = \{\langle d, d \rangle \in \mathcal{D}_\mathfrak{M}\}$;
let g be a variable assignment (a mapping that assigns to each free variable x some member $g(x)$ of the model domain $\mathcal{D}_\mathfrak{M}$) and let $I^g_{\langle w,t \rangle}$ be a function which assigns an extension to each pair of an atomic predicate, a modified predicate or a term of $\mathcal{L}$ and a pair $\langle w, t \rangle$, where $w \in W, t \in T$:
– if x a variable, then $I^g_{\langle w,t \rangle}(x) = g(x)$ for any $\langle w, t \rangle$;
– $I \subseteq I^g$ for any g;
the notion of interpretation of terms other than variables and interpretation of modified predicates and satisfaction of formulas in $\mathfrak{M}$ are defined as follows:

$S1$. if Q is a n-place predicate constant and $y_1, \dots, y_n$ are variables, then $\mathfrak{M}^{g\,w\,t} \vDash Q(y_1, \dots, y_n)$ iff $\langle g(y_1), \dots, g(y_n) \rangle \in I_{\langle w,t \rangle}(Q)$; the notions of satisfaction of $\sim\varphi$ ($S2$), $(\varphi \wedge \psi)$ ($S3$), $(\varphi \vee \psi)$ ($S4$), $(\varphi \rightarrow \psi)$ ($S5$), $(\varphi \leftrightarrow \psi)$ ($S6$) are defined in a standard way;

6 This definition is taken after Fitting & Mendelsohn (1998, p. 103 Definition 4.7.3). I accept the authors' reasoning behind it.

$S7.$ if Q is a n-place predicate constant, P is a 1-place predicate constant and x is a variable, then

$I_{\langle w,t\rangle}(Q^i_{\lambda x.P(x)}) \in \mathcal{P}(\{\langle d_1, \dots, d_i, \dots, d_n\rangle \in I_{\langle w,t\rangle}(Q) : d_i \in I_{\langle w,t\rangle}(P)\})$;

$S8.$ if $Q^i_{\lambda x.P(x)}$, $Q^i_{\lambda y.P(y)}$ are n-place atomic predicates modified by $\lambda x.P(x)$, $\lambda y.P(y)$ on ith argument place and x, y are variables, then $I_{\langle w,t\rangle}(Q^i_{\lambda x.P(x)}) = I_{\langle w,t\rangle}(Q^i_{\lambda y.P(y)})$;

$S9.$ if Q is a n-place predicate constant, x is a variable, and $(\lambda x.\varphi)$, $(\lambda x.\psi)$ are modifiers, then $I_{\langle w,t\rangle}(Q^i_{\lambda x.(\varphi\wedge\psi)}) = I_{\langle w,t\rangle}(Q^i_{\lambda x.\varphi}) \cap I_{\langle w,t\rangle}(Q^i_{\lambda x.\psi})$;

$S10.$ if $Q(z_1, \dots, z_n)$ is an atomic formula and $(\lambda x.Q(z_1, \dots, z_n))^i_{\lambda y.\psi}$ is a modified predicate abstract, then

$I^g_{\langle w,t\rangle}\left((\lambda x.Q(z_1, \dots, z_n))^i_{\lambda y.\psi}\right) = \left\{d \in \mathcal{D}_{\mathfrak{M}} : \mathfrak{M}^{g\,\binom{d}{x} w\, t} \vDash Q^i_{\lambda y.\psi}(z_1, \dots, z_n)\right\}$;

$S11.$ if $\sim Q(z_1, \dots, z_n)$ is a negation of an atomic formula and $(\lambda x.\sim Q(z_1, \dots, z_n))^i_{\lambda y.\psi}$ is a modified predicate abstract, then

$I^g_{\langle w,t\rangle}\left((\lambda x.\sim Q(z_1, \dots, z_n))^i_{\lambda y.\psi}\right) = \left\{d \in \mathcal{D}_{\mathfrak{M}} : \mathfrak{M}^{g\,\binom{d}{x} w\, t} \nvDash Q^i_{\lambda y.\psi}(z_1, \dots, z_n)\right\}$;

$S12.$ if Q is a n-place predicate constant, $(\lambda x.\varphi)$ is a modifier and $Q^i_{\lambda x.\varphi}$ is a n-place modified predicate, then $\mathfrak{M}^{g\,w\,t} \vDash Q^i_{\lambda x.\varphi}(z_1, \dots, z_n)$ iff $\langle g(z_1), \dots, g(z_n)\rangle \in I_{\langle w,t\rangle}(Q^i_{\lambda x.\varphi})$; the notions of satisfaction $\mathbf{P}\varphi$ ($S13$), $\mathbf{F}\varphi$ ($S14$) are defined in a standard way;

$S15.$ if φ is a formula, then $\mathfrak{M}^{g\,w\,t_j} \vDash [_i]\varphi$ iff $\mathfrak{M}^{g\,w\,t_i} \vDash \varphi$; the notions of satisfaction $\Box\varphi$ ($S16$), $\Diamond\varphi$ ($S17$), $\forall_x\varphi$ ($S18$), $\exists_x\varphi$ ($S19$) are defined in a standard way;

$S20.$ if $\mathfrak{M}^{g\,\binom{d}{x} w\, t} \vDash \varphi$ for exactly one $d \in \mathcal{D}_{\mathfrak{M}}$, then $I^g_{\langle w,t\rangle}(\iota x.\varphi) = d$; if it is not the case that $\mathfrak{M}^{g\,\binom{d}{x} w\, t} \vDash \varphi$ for exactly one $d \in \mathcal{D}_{\mathfrak{M}}$, then $\iota x.\varphi$ fails to designate at $\langle w, t\rangle$ in $\mathfrak{M}$ with respect to g; the notion of satisfaction of $(\lambda x.\varphi)(s)$ ($S21$) is defined in a standard way;

$S22.$ if a term s designates at $\langle w, t\rangle$ in $\mathfrak{M}$ with respect to g and $(\lambda x.\varphi)^i_{\lambda y.\psi}$ is a modified predicate abstract, then $\mathfrak{M}^{g\,w\,t} \vDash (\lambda x.\varphi)^i_{\lambda y.\psi}(s)$ iff $I^g_{\langle w,t\rangle}(s) \in I^g_{\langle w,t\rangle}((\lambda x.\varphi)^i_{\lambda y.\psi})$; if a term s fails to designate at $\langle w, t\rangle$ in $\mathfrak{M}$ with respect to g, then $\mathfrak{M}^{g\,w\,t} \nvDash (\lambda x.\varphi)^i_{\lambda y.\psi}(s)$.

I will use symbol "$!x.\varphi$" for a special case of $\iota x.\varphi$ terms with only one variable x which occurs free in φ. There are no free variable occurrences in $!x.\varphi$ and due to this if $I^{g}_{\langle w,t\rangle}(!x.\varphi)$ is defined then $I^{g}_{\langle w,t\rangle}(!x.\varphi) = I^{g'}_{\langle w,t\rangle}(!x.\varphi)$ for any assignments g and g'. That is why instead of "$I^{g}_{\langle w,t\rangle}(!x.\varphi)$" we will write "$I_{\langle w,t\rangle}(!x.\varphi)$" which should be understood as "$I^{g}_{\langle w,t\rangle}(!x.\varphi)$" where g is any assignment.

Now I will expand language $\mathcal{L}$ to $\mathcal{L}^{+}$ by adding name-term and pseudonym-terms. I will skip all syntactical and semantic definitions of $\mathcal{L}^{+}$ duplicating the definitions of $\mathcal{L}$ and will write below only new ones.

Definition IV*: The alphabet of* $\mathcal{L}^{+}$

A first-order language $\mathcal{L}^{+}$ contains all symbols of $\mathcal{L}$ with the addition of an infinite set of name-terms $\mathcal{N} = \{n_1, n_2, n_3, \dots\}$ and an infinite set of pseudonym-terms $\mathcal{M} = \{m_1, m_2, m_3, \dots\}$.

Definition V*: The syntax of* $\mathcal{L}^{+}$

$R1$. the same as $R1$. of $\mathcal{L}$;

$R2$. a name-term or a pseudonym-term is a term with no free variable occurrences;

$R3$. – $R12$. are the same as $R2$. – $R11$. of $\mathcal{L}$;

$R13$. s is a modifier, where s is a name-term or a pseudonym-term;

$R14$. – $R16$. are the same as $R12$. – $R14$. of $\mathcal{L}$;

$R17$. if $(\lambda x.Q(z_1, \dots, z_n))$ is a predicate abstract and s is a name-term or a pseudonym-term, then $(\lambda x.Q(z_1, \dots, z_n))^{i}_{s}$ is a predicate abstract modified by s on ith argument place of Q (where $1 \leq i \leq n$); the free variable occurrences in $(\lambda x.Q(z_1, \dots, z_n))^{i}_{s}$ are those of $(\lambda x.Q(z_1, \dots, z_n))$;

$R18$. if $(\lambda x.{\sim}Q(z_1, \dots, z_n))$ is a predicate abstract and s is a name-term or a pseudonym-term, then $(\lambda x.{\sim}Q(z_1, \dots, z_n))^{i}_{s}$ is a predicate abstract modified by s on ith argument place of Q (where $1 \leq i \leq n$); the free variable occurrences in $(\lambda x.{\sim}Q(z_1, \dots, z_n))^{i}_{s}$ are those of $(\lambda x.{\sim}Q(z_1, \dots, z_n))$;

$R19$.– $R21$. are the same as $R16$. – $R18$. of $\mathcal{L}$;

$R22.$ if $(\lambda x.\varphi)^i_{s_j}$ is a modified predicate abstract and s_k is a name-term or a pseudonym-term, then $(\lambda x.\varphi)^i_{s_j}(s_k)$ is a formula iff $k = j$; the free variable occurrences in $(\lambda x.\varphi)^i_{s_j}(s_k)$ are those of $(\lambda x.\varphi)$;

$R23.$ the same as $R19.$ of $\mathcal{L}$.

Definition VI*: The semantics of* $\mathcal{L}^+$

Let $\mathfrak{M} = \langle \mathcal{D}, T, <, W, I \rangle$ be a model of $\mathcal{L}$. A varying domain first-order model $\mathfrak{M}^{\leq}$ for $\mathcal{L}^+$ is a structure $\mathfrak{M}^{\leq} = \langle \mathcal{D}, T, <, W, I^{\leq} \rangle$, where $I^{\leq} \restriction \mathcal{L} = I$.

Using already defined properties of $\mathfrak{M}$ (*Definition* III) we define the following sets, relations and functions.

$S(a)$: set $\Gamma_{\mathcal{L}}$

Set $\Gamma_{\mathcal{L}}$ is a set of iota-terms $!x.[_i]\varphi$ of $\mathcal{L}$. $!x.[_i]\varphi \in \Gamma_{\mathcal{L}}$ iff 1) there is a world $w \in W$ such that for every time $t \in T$ $!x.[_i]\varphi$ designates at $\langle w, t \rangle$ in $\mathfrak{M}$; 2) $\varphi = \big(N_i(x) \wedge P(x)\big)$ or $\varphi = \Big(N_{i\,\lambda y.Q(y)}(x) \wedge Q(x)\Big)$ or $\varphi = \Big(N_{i\,\lambda y.Q(y)}(x) \wedge P_{\lambda y.Q(y)}(x)\Big)$, where N_i is a distinguished predicate and P, Q are undistinguished predicates. (I will use symbols "γ_i", "γ_j"for members of $\Gamma_{\mathcal{L}}$)

$S(b)$: set Δ

$\Delta \subseteq \Gamma_{\mathcal{L}} \times W$. $\langle \gamma_i, w \rangle \in \Delta$ iff for any time $t \in T$ $I_{\langle w,t \rangle}(\gamma_i)$ is defined.

$S(c)$: set $\boldsymbol{D}$

$\boldsymbol{D} \subseteq \Delta$. $\langle \gamma_i, w \rangle \in \boldsymbol{D}$ iff there is a predicate $N_{i\,\lambda y.Q(y)}$ and a time $t \in T$ such that $I_{\langle w,t \rangle}(\gamma_i) \in I_{\langle w,t \rangle}\big(N_{i\,\lambda y.Q(y)}\big)$ and γ_i contains Q or N_i, where N_i is a distinguished predicate and Q is an undistinguished predicate.

$S(d)$: set $\boldsymbol{D}^*$

$\boldsymbol{D}^* \subseteq \boldsymbol{D}$. $\langle !x.[_i]\varphi, w \rangle \in \boldsymbol{D}^*$ iff $\varphi = \Big(N_{i\,\lambda y.Q(y)}(x) \wedge Q(x)\Big)$ or $\varphi = \Big(N_{i\,\lambda y.Q(y)}(x) \wedge P_{\lambda y.Q(y)}(x)\Big)$, where N_i is a distinguished predicate and P, Q are undistinguished predicates.

Let $\Delta^* = \Delta \setminus (\boldsymbol{D} \setminus \boldsymbol{D}^*)$.

$S(e)$: *relation* $\mathbb{R}$
$\mathbb{R} \subseteq \Delta^{*2}$. $\langle\gamma_i, w\rangle\mathbb{R}\langle\gamma_j, w'\rangle$ iff for any time $t \in T$ $I_{\langle w,t\rangle}(\gamma_i) = I_{\langle w',t\rangle}(\gamma_j)$ and there is either the same predicate N_k or the same predicate $N_{k\ \lambda y.Q(y)}$ in γ_i, γ_j.

Let $\Delta^* / \mathbb{R}$ be a partition of set Δ^* by equivalence relation $\mathbb{R}$ and $[\langle\gamma_i, w\rangle]_{\mathbb{R}}$ be an equivalence class from $\Delta / \mathbb{R}$.

$S(f)$: *function* $\mathbb{F}$
$\mathbb{F}$: $\Delta^* / \mathbb{R} \rightarrow \mathcal{D}_{\mathfrak{M}}$. For any $[\langle\gamma_i, w\rangle]_{\mathbb{R}} \in \Delta^* / \mathbb{R}$, $\mathbb{F}([\langle\gamma_i, w\rangle]_{\mathbb{R}}) = d$, where for any time $t \in T$ $d = I_{\langle w,t\rangle}(\gamma_j)$ for any $\langle\gamma_j, w\rangle \in [\langle\gamma_i, w\rangle]_{\mathbb{R}}$.

Let $\leq$ be any well-order relation on a set $\Delta^* / \mathbb{R}$ and let $\langle\Delta^*/\mathbb{R}, \leq\rangle$ be well-ordered set.

$S(g)$: *function* $\mathbb{Q}^{\leq}$
$\mathbb{Q}^{\leq}$: $\{\mathcal{N} \cup \mathcal{M}\} \rightarrow \Delta^* / \mathbb{R}$. Function $\mathbb{Q}^{\leq}$ for an argument gives an equivalence class $[\langle\gamma_i, w\rangle]_{\mathbb{R}}$ in the following way:
– for n_1 $\mathbb{Q}^{\leq}$ gives the least element of $\langle(\Delta^* / \boldsymbol{D})/\mathbb{R}, \leq \cap ((\Delta^* / \boldsymbol{D})/\mathbb{R})^2\rangle$;
– for every next element of $\mathcal{N}$ (with respect to an index) $\mathbb{Q}^{\leq}$ gives next element of $\langle(\Delta^* / \boldsymbol{D})/\mathbb{R}, \leq \cap ((\Delta^* / \boldsymbol{D})/\mathbb{R})^2\rangle$;
– in case there are no next element in $\langle(\Delta^* / \boldsymbol{D})/\mathbb{R}, \leq \cap ((\Delta^* / \boldsymbol{D})/\mathbb{R})^2\rangle$ then for a next element of $\mathcal{N}$ $\mathbb{Q}^{\leq}$ gives the least element of $\langle(\Delta^* / \boldsymbol{D})/\mathbb{R}, \leq \cap ((\Delta^* / \boldsymbol{D})/\mathbb{R})^2\rangle$;

– for m_1 $\mathbb{Q}^{\leq}$ gives the least element of $\langle\boldsymbol{D}^*/\mathbb{R}, \leq \cap (\boldsymbol{D}^*/\mathbb{R})^2\rangle$;
– for every next element of $\mathcal{M}$ (with respect to an index) $\mathbb{Q}^{\leq}$ gives next element of $\langle\boldsymbol{D}^*/\mathbb{R}, \leq \cap (\boldsymbol{D}^*/\mathbb{R})^2\rangle$;
– in case there are no next element in $\langle\boldsymbol{D}^*/\mathbb{R}, \leq \cap (\boldsymbol{D}^*/\mathbb{R})^2\rangle$ then for a next element of $\mathcal{M}$ $\mathbb{Q}^{\leq}$ gives the least element of $\langle\boldsymbol{D}^*/\mathbb{R}, \leq \cap (\boldsymbol{D}^*/\mathbb{R})^2\rangle$.

$S(h)$: *relation* $\mathbb{S}$
$\mathbb{S} \subseteq \Delta^{*2}$. $\langle\gamma_i, w\rangle\mathbb{S}\langle\gamma_j, w\rangle$ iff $\langle\gamma_i, w\rangle$, $\langle\gamma_j, w'\rangle$ belong to the same equivalence class $[\langle\gamma_i, w\rangle]_{\mathbb{R}}$ and $w = w'$.

$S(i)$: *function* $h^{\leq}$
$h^{\leq}: \{\mathcal{N} \cup \mathcal{M}\} \times W \rightarrow \Delta^* \,/\, \mathbb{S}$. For any $n_i \in \mathcal{N}$, $w \in W$ $h^{\leq}(n_i, w) = [\langle \gamma_j, w \rangle]_{\mathbb{S}} \subseteq \mathbb{Q}^{\leq}(n_i)$ if there is such an equivalence class, otherwise $h^{\leq}(n_i, w)$ is undefined. For any $m_j \in \mathcal{M}$, $w \in W$ $h^{\leq}(m_j, w) = [\langle \gamma_i, w \rangle]_{\mathbb{S}} \subseteq \mathbb{Q}^{\leq}(m_j)$ if there is such an equivalence class, otherwise $h^{\leq}(m_j, w)$ is undefined.

Semantic rules $S1. - S20.$ of language $\mathcal{L}^+$ are the same as rules $S1. - S20.$ of language $\mathcal{L}$ (except of talking about $I^{\leq}$ instead of I);

$S21.$ if n_i is a name-term and $\Delta^* \,/\, \boldsymbol{D} \neq \emptyset$, then $I^{\leq}_{\langle w,t \rangle}(n_i) = \mathbb{F}(\mathbb{Q}^{\leq}(n_i))$; if $\Delta^* \,/\, \boldsymbol{D} = \emptyset$, then n_i fails to designate in $\mathfrak{M}^{\leq}$ (at any $\langle w', t' \rangle$);

$S22.$ if m_i is a pseudonym-term and $\boldsymbol{D}^* \neq \emptyset$, then $I^{\leq}_{\langle w,t \rangle}(m_i) = \mathbb{F}(\mathbb{Q}^{\leq}(m_i))$; if $\boldsymbol{D}^* = \emptyset$, then m_i fails to designate in $\mathfrak{M}^{\leq}$ (at any $\langle w', t' \rangle$);

$S23.$ if a term s designates at $\langle w, t \rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\, w\, t} \vDash (\lambda x. \varphi)(s)$ iff $\mathfrak{M}^{\leq g\, (^{d}_{x}) w\, t} \vDash \varphi$, where $d = I^{\leq g}_{\langle w,t \rangle}(s)$; if a term s fails to designate at $\langle w, t \rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\, w\, t} \nvDash (\lambda x. \varphi)(s)$;

$S24.$ if a term s designates at $\langle w, t \rangle$ in $\mathfrak{M}^{\leq}$ with respect to g and $(\lambda x. \varphi)^{i}_{\lambda y. \psi}$ is a modified predicate abstract, then $\mathfrak{M}^{\leq g\, w\, t} \vDash (\lambda x. \varphi)^{i}_{\lambda y. \psi}(s)$ iff $I^{g}_{\langle w,t \rangle}(s) \in I^{g}_{\langle w,t \rangle}\big((\lambda x. \varphi)^{i}_{\lambda y. \psi}\big)$; if a term s fails to designate at $\langle w, t \rangle$ in $\mathfrak{M}^{\leq}$ with respect to g, then $\mathfrak{M}^{\leq g\, w\, t} \nvDash (\lambda x. \varphi)^{i}_{\lambda y. \psi}(s)$;

$S25.$ if n_k is a name-term and $(\lambda x. \varphi)^{i}_{n_k}$ is a predicate abstract modified by n_k, then $\mathfrak{M}^{\leq g\, w\, t_j} \vDash (\lambda x. \varphi)^{i}_{n_k}(n_k)$ iff there is a description $!y. [_j]\psi \in \pi_1\big(h^{\leq}(n_k, w)\big)$, such that $\mathfrak{M}^{\leq g\, w\, t_j} \vDash (\lambda x. \varphi)^{i}_{\lambda y. \psi}(n_k)$;

$S26.$ if m_k is a pseudonym-term and $(\lambda x. \varphi)^{i}_{m_k}$ is a predicate abstract modified by m_k, then $\mathfrak{M}^{\leq g\, w\, t_j} \vDash (\lambda x. \varphi)^{i}_{m_k}(m_k)$ iff there is a description $!y. [_j]\psi \in \pi_1\big(h^{\leq}(m_k, w)\big)$, such that $\lambda z. Q(z)$ is a modifier of a predicate $N_{i\, \lambda z. Q(z)}$ from the description $!y. [_j]\psi$ and $\mathfrak{M}^{\leq g\, w\, t_j} \vDash (\lambda x. \varphi)^{i}_{\lambda y. \psi}(m_k)$.

In (Poller 2016) I have proven that you can take a modifier "in and out" of a predicate abstracted from an atomic formula or a negation of atomic formula,
$\mathfrak{M}^{\leq g\ w\ t} \vDash \left(\lambda x.\, Q^{i}_{\lambda y.\psi}\,(z_1, \dots, z_n)\right)(s)$
iff $\mathfrak{M}^{\leq g\ w\ t} \vDash \left(\lambda x.\, Q(z_1, \dots, z_n)\right)^{i}_{\lambda y.\psi}(s)$,
$\mathfrak{M}^{\leq g\ w\ t} \vDash \left(\lambda x.\, {\sim}Q^{i}_{\lambda y.\psi}\,(z_1, \dots, z_n)\right)(s)$
iff $\mathfrak{M}^{\leq g\ w\ t} \vDash \left(\lambda x.\, {\sim}Q(z_1, \dots, z_n)\right)^{i}_{\lambda y.\psi}(s)$,
which is very useful in proofs (I will refer to it as Theorem). Now I will model a sentence from The Puzzle.

Let $\mathfrak{M}^{\leq}$ be a model of $\mathcal{L}^{+}$, $W = \{w\}$, $\mathcal{D}_{\langle w,t_1\rangle} = \{$ ♟ , ♙ , [suːpəmæn], [klaːk kɛnt], [ləʊ. ɪs]$\}$, $\mathcal{D}_{\langle w,t_j\rangle} = \emptyset$ for $j \neq 1$. Let us use symbols "R" ("reporter"), "S" ("superhero"), "P" ("talks on the phone with"), "L" ("look through the window at") instead "P_1", "P_2", "P_3", "P_4" of $\mathcal{L}^{+}$. Let use symbol "N_1" for "called [klaːk kɛnt]", symbol "N_2" for "called [suːpəmæn]" and symbol "N_3" for "called [ləʊ. ɪs]". Let $I^{\leq}$ be defined in following way:

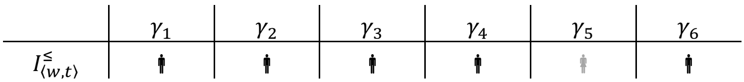
	S	R	P	L	P_i $i \geq 5$	N_1	N_2	N_3	N_i $i \neq 1$
$I^{\leq}_{\langle w,t_1\rangle}$	{ ♟ }	{ ♟ , ♙ }	⟨ ♟ , ♙ ⟩ ⟨ ♙ , ♟ ⟩	⟨ ♟ , ♙ ⟩ ⟨ ♙ , ♟ ⟩	$\emptyset$	{ ♟ }	{ ♟ }	{ ♙ }	$\emptyset$
$I^{\leq}_{\langle w,t_i\rangle}$ $i \neq 1$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

	$P^2_{N_1}$	P^2_S	L^2_S	L^2_R	$L^2_{N_1}$	$N_{2\,S}$
$I^{\leq}_{\langle w,t_1\rangle}$	$\emptyset$	⟨ ♙ , ♟ ⟩	$\emptyset$	⟨ ♙ , ♟ ⟩	⟨ ♙ , ♟ ⟩	{ ♟ }
$I^{\leq}_{\langle w,t_i\rangle}$ $i \neq 1$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$	$\emptyset$

For a predicate Q and any time $I^{\leq}_{\langle w,t\rangle}(Q_Q) = I^{\leq}_{\langle w,t\rangle}(Q)$. For predicates other than those mentioned above and $\langle w, t\rangle$, where t is any time, function $I^{\leq}$ gives $\emptyset$.

Set $\Gamma_{\mathcal{L}}$ (*Def.* VI. $S(a)$):

γ_1	$!\,x.\,[_1]\big(S(x) \wedge N_1(x)\big)$	γ_4	$!\,x.\,[_1]\big(R(x) \wedge N_2(x)\big)$
γ_2	$!\,x.\,[_1]\big(S(x) \wedge N_2(x)\big)$	γ_5	$!\,x.\,[_1]\big(R(x) \wedge N_3(x)\big)$
γ_3	$!\,x.\,[_1]\big(R(x) \wedge N_1(x)\big)$	γ_6	$!\,x.\,[_1]\big(S(x) \wedge N_{2\,S}(x)\big)$



Set Δ (*Def.* VI. $S(b)$)

$\langle\gamma_1, w\rangle$ $\langle\gamma_3, w\rangle$ $\langle\gamma_5, w\rangle$
$\langle\gamma_2, w\rangle$ $\langle\gamma_4, w\rangle$ $\langle\gamma_6, w\rangle$

Set $\boldsymbol{D}$ (*Def.* VI. $S(c)$)

$\langle\gamma_1, w\rangle$ $\langle\gamma_4, w\rangle$
$\langle\gamma_2, w\rangle$ $\langle\gamma_6, w\rangle$

Set $\boldsymbol{D}^*$ (*Def.* VI. $S(d)$)

$\langle\gamma_6, w\rangle$

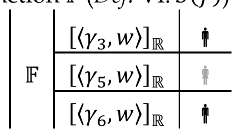
Set Δ^* (*Def.* VI. $S(e)$)

$\langle\gamma_3, w\rangle$ $\langle\gamma_5, w\rangle$ $\langle\gamma_6, w\rangle$

Δ^* / $\mathbb{R}$ (*Def.* VI. $S(e)$)

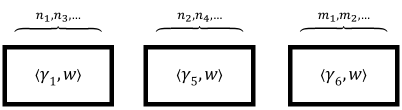
$\langle\gamma_3, w\rangle$ | $\langle\gamma_5, w\rangle$ | $\langle\gamma_6, w\rangle$

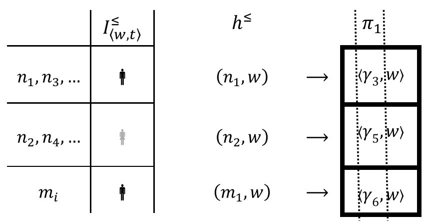
Function $\mathbb{F}$ (*Def.* VI. $S(f)$)



Function $\mathbb{Q}^{\leq}$ (*Def.* VI. $S(f)$): $I^{\leq}_{\langle w,t\rangle}(s) = \mathbb{F}\big(\mathbb{Q}^{\leq}(s)\big)$, Function $h^{\leq}$ (*Def.* VI. $S(i)$)

where s is n_j or m_i





Let us see what the value of the following sentences is:

(*a*) While talking on the phone to Superman (as Superman), Lois looked through the window at Clark Kent (as Clark Kent);

(*b*) While talking on the phone to Clark Kent (as Clark Kent), Lois looked through the window at Superman (as Superman).

$$(a)\ \lambda y.\left(\left(\lambda x.P(y,x)\right)^2_{m_1}(m_1)\right)(n_2) \wedge \lambda y.\left(\left(\lambda x.L(y,x)\right)^2_{n_1}(n_1)\right)(n_2);$$

$$(b)\ \lambda y.\left(\left(\lambda x.P(y,x)\right)^2_{n_1}(n_1)\right)(n_2) \wedge \lambda y.\left(\left(\lambda x.L(y,x)\right)^2_{m_1}(m_1)\right)(n_2).$$

(*a*)

$\mathfrak{M}^{\leq g\,w\,t_1} \vDash \lambda y.\left(\left(\lambda x.P(y,x)\right)^2_{m_1}(m_1)\right)(n_2) \wedge \lambda y.\left(\left(\lambda x.L(y,x)\right)^2_{n_1}(n_1)\right)(n_2)$

iff (Def. VI.$S3$)

$\mathfrak{M}^{\leq g\,w\,t_1} \vDash \lambda y.\left(\left(\lambda x.P(y,x)\right)^2_{m_1}(m_1)\right)(n_2)$ and

$\mathfrak{M}^{\leq g\,w\,t_1} \vDash \lambda y.\left(\left(\lambda x.L(y,x)\right)^2_{n_1}(n_1)\right)(n_2)$ iff (*Def.* VI.$S23$)

$\mathfrak{M}^{\leq g\binom{d}{y}w\,t_1} \vDash \left(\lambda x.P(y,x)\right)^2_{m_1}(m_1)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ and

$\mathfrak{M}^{\leq g\binom{e}{y}w\,t_1} \vDash \left(\lambda x.L(y,x)\right)^2_{n_1}(n_1)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ iff (*Def.* VI.$S26, S25$)

there is a description $!y.[_1]\varphi \in \pi_1(h^{\leq}(m_1,w))$, such that $\lambda z.Q(z)$ is a modifier of a predicate $N_{i\,\lambda z.Q(z)}$ from the description $!y.[_1]\varphi$ and

$\mathfrak{M}^{\leq g\binom{d}{y}w\,t_1} \vDash \left(\lambda x.P(y,x)\right)^2_{\lambda z.Q(z)}(m_1)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ and

there is a description $!z.[_1]\psi \in \pi_1\left(h^{\leq}(n_1,w)\right)$,

such that $\mathfrak{M}^{\leq g\binom{e}{y}w\,t_1} \vDash \left(\lambda x.L(y,x)\right)^2_{\lambda z.\psi}(n_1)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ iff (Theorem)

there is a description $!y.[_1]\varphi \in \pi_1\left(h^{\leq}(m_1,w)\right)$, such that $\lambda z.Q(z)$ is a modifier of a predicate $N_{i\,\lambda z.Q(z)}$ from the description $!y.[_1]\varphi$ and

$\mathfrak{M}^{\leq g\binom{d}{y}w\,t_1} \vDash \left(\lambda x.P^2{}_{\lambda z.Q(z)}(y,x)\right)(m_1)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ and

there is a description $!z.[_1]\psi \in \pi_1\left(h^{\leq}(n_1,w)\right)$,

such that $\mathfrak{M}^{\leq g\binom{e}{y}w\,t_1} \vDash \left(\lambda x.L^2_{\lambda z.\psi}(y,x)\right)(n_1)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ iff (*Def.* VI.$S23$)

there is a description $!y.[_1]\varphi \in \pi_1\left(h^{\leq}(m_1,w)\right)$, such that $\lambda z.Q(z)$ is a modifier of a predicate $N_{i\,\lambda z.Q(z)}$ from the description $!y.[_1]\varphi$ and

$\mathfrak{M}^{\leq g\binom{d}{y}\binom{d_1}{x}w\,t_1} \vDash P^2{}_{\lambda z.Q(z)}(y,x)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $d_1 = I^{\leq}_{\langle w,t_1\rangle}(m_1)$ and there is a description $!z.[_1]\psi \in \pi_1(h^{\leq}(n_1, w))$, such that $\mathfrak{M}^{\leq g \binom{e}{y}\binom{e_1}{x}\, w\, t_1} \vDash L^2_{\lambda z.\psi}(y, x)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $e_1 = I^{\leq}_{\langle w,t_1\rangle}(n_1)$ iff (*Def.* VI.S12) there is a description $!y.[_1]\varphi \in \pi_1(h^{\leq}(m_1, w))$, such that $\lambda z.Q(z)$ is a modifier of a predicate $N_{i\ \lambda z.Q(z)}$ from the description $!y.[_1]\varphi$ and $\langle d, d_1\rangle \in I^{\leq}_{\langle w,t_1\rangle}(P^2{}_{\lambda z.Q(z)})$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $d_1 = I^{\leq}_{\langle w,t_1\rangle}(m_1)$ and there is a description $!z.[_1]\psi \in \pi_1(h^{\leq}(n_1, w))$, such that $\langle e, e_1\rangle \in I^{\leq}_{\langle w,t_1\rangle}(L^2_{\lambda z.\psi})$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $e_1 = I^{\leq}_{\langle w,t_1\rangle}(n_1)$.

It is so that $I^{\leq}_{\langle w,t_1\rangle}(n_2) = d = e =$, $I^{\leq}_{\langle w,t_1\rangle}(m_1) = d_1 =$, $I^{\leq}_{\langle w,t_1\rangle}(n_1) = e_1 =$.
Let $!z.[_1]\psi \in \pi_1(h^{\leq}(n_1, w))$ be $!x.[_1](R(x) \wedge N_1(x)), \gamma_3$.
It is so that ⟨ , ⟩ $\in I^{\leq}_{\langle w,t_1\rangle}(L^2_R)$, ⟨  , ⟩ $\in I^{\leq}_{\langle w,t_1\rangle}(L^2_{N_1})$,
so (*Def.* VI.S9) , ⟨ , ⟩ $\in I^{\leq}_{\langle w,t_1\rangle}\left(L^2_{\lambda x.(R(x)\wedge N_1(x))}\right)$. Modifier $\lambda x.S(x)$ is a modifier of a predicate $N_{i\ \lambda z.Q(z)}$ from any description $!y.[_1]\varphi \in \pi_1(h^{\leq}(m_1, w))$. It is so that ⟨ , ⟩ $\in I^{\leq}_{\langle w,t_1\rangle}(P^2_S)$. This means that formula (a) is satisfied.

(b)

$\mathfrak{M}^{\leq g\, w\, t_1} \vDash \lambda y.\left(\left(\lambda x.P(y,x)\right)^2_{n_1}(n_1)\right)(n_2) \wedge \lambda y.\left(\left(\lambda x.L(y,x)\right)^2_{m_1}(m_1)\right)(n_2)$ iff

(*Def.* VI.S3)

$\mathfrak{M}^{\leq g\, w\, t_1} \vDash \lambda y.\left(\left(\lambda x.P(y,x)\right)^2_{n_1}(n_1)\right)(n_2)$ and
$\mathfrak{M}^{\leq g\, w\, t_1} \vDash \lambda y.\left(\left(\lambda x.L(y,x)\right)^2_{m_1}(m_1)\right)(n_2)$ iff (*Def.* VI.S23)
$\mathfrak{M}^{\leq g \binom{d}{y} w\, t_1} \vDash \left(\lambda x.P(y,x)\right)^2_{n_1}(n_1)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ and
$\mathfrak{M}^{\leq g \binom{e}{y} w\, t_1} \vDash \left(\lambda x.L(y,x)\right)^2_{m_1}(m_1)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ iff (*Def.* VI.$S25, S26$)
there is a description $!z.[_1]\psi \in \pi_1(h^{\leq}(n_1, w))$, such that $\mathfrak{M}^{\leq g \binom{d}{y} w\, t_1} \vDash \left(\lambda x.P(y,x)\right)^2_{\lambda z.\psi}(n_1)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ and there is a description $!y.[_1]\varphi \in \pi_1(h^{\leq}(m_1, w))$, such that $\lambda z.Q(z)$ is a modifier of a predicate $N_{i\ \lambda z.Q(z)}$ from the description $!y.[_1]\varphi$ and $\mathfrak{M}^{\leq g \binom{e}{y} w\, t_1} \vDash \left(\lambda x.L(y,x)\right)^2_{\lambda z.Q(z)}(m_1)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ iff (*Theorem*) there is a description $!z.[_1]\psi \in \pi_1(h^{\leq}(n_1, w))$, such that $\mathfrak{M}^{\leq g \binom{d}{y} w\, t_1} \vDash \left(\lambda x.P^2{}_{\lambda z.\psi}(y,x)\right)(n_1)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ and there is a description $!y.[_1]\varphi \in \pi_1(h^{\leq}(m_1, w))$, such that $\lambda z.Q(z)$ is a modifier of a predicate $N_{i\ \lambda z.Q(z)}$ from the description $!y.[_1]\varphi$ and $\mathfrak{M}^{\leq g \binom{e}{y} w\, t_1} \vDash \left(\lambda x.L^2{}_{\lambda z.Q(z)}(y,x)\right)(m_1)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$ iff (*Def.* VI.S23) there is a description $!z.[_1]\psi \in \pi_1(h^{\leq}(n_1, w))$, such that

$\mathfrak{M}^{\leq g\binom{d}{y}\binom{d_1}{x} w\, t_1} \vDash P^2{}_{\lambda z.\psi}(y,x)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $d_1 = I^{\leq}_{\langle w,t_1\rangle}(n_1)$ and there is a description $!\,y.\,[_1]\varphi \in \pi_1\big(h^{\leq}(m_1,w)\big)$, such that $\lambda z.\,Q(z)$ is a modifier of a predicate $N_{i\ \lambda z.Q(z)}$ from the description $!\,y.\,[_1]\varphi$ and $\mathfrak{M}^{\leq g\binom{e}{y}\binom{e_1}{x} w\, t_1} \vDash L^2{}_{\lambda z.Q(z)}(y,x)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $e_1 = I^{\leq}_{\langle w,t_1\rangle}(m_1)$ iff (*Def.* VI.S12)
there is a description $!\,z.\,[_1]\psi \in \pi_1\big(h^{\leq}(n_1,w)\big)$, such that $\langle d, d_1\rangle \in I^{\leq}_{\langle w,t_1\rangle}\big(P^2{}_{\lambda z.\psi}\big)$, where $d = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $d_1 = I^{\leq}_{\langle w,t_1\rangle}(n_1)$ and there is a description $!\,y.\,[_1]\varphi \in \pi_1\big(h^{\leq}(m_1,w)\big)$, such that $\lambda z.\,Q(z)$ is a modifier of a predicate $N_{i\ \lambda z.Q(z)}$ from the description $!\,y.\,[_1]\varphi$ and $\langle e, e_1\rangle \in I^{\leq}_{\langle w,t_1\rangle}\big(L^2_{\lambda z.Q(z)}\big)$, where $e = I^{\leq}_{\langle w,t_1\rangle}(n_2)$, $e_1 = I^{\leq}_{\langle w,t_1\rangle}(m_1)$.

It is so that $I^{\leq}_{\langle w,t_1\rangle}(n_2) = d = e =$, $I^{\leq}_{\langle w,t_1\rangle}(m_1) = e_1 =$, $I^{\leq}_{\langle w,t_1\rangle}(n_1) = d_1 =$. Every description from the set $\pi_1\big(h^{\leq}(n_1,w)\big)$ contains the predicate N_1. It is so that $I^{\leq}_{\langle w,t_1\rangle}\big(P^2_{N_1}\big) = \emptyset$, which means that ⟨ , ⟩ $\notin I^{\leq}_{\langle w,t_1\rangle}\big(P^2_{N_1}\big)$. This in turn means that for any description $!\,z.\,[_1]\psi \in \pi_1\big(h^{\leq}(n_1,w)\big)$ ⟨ , ⟩ $\notin I^{\leq}_{\langle w,t_1\rangle}\big(P^2_{\lambda z.\psi}\big)$ (*Def.* VI.S9). The first part of (b) formula's conjunction is not satisfied, so (b) is n o t s a t i s f i e d .

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