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Rozprawa

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PHENOMENOLOGICAL IDEAS IN THE PHILOSOPHY OF MATHEMATICS. FROM HUSSERL TO GÖDEL

Summary: The paper is devoted to phenomenological ideas in conceptions of modern philosophy of mathematics. Views of Husserl, Weyl, Becker and Gödel will be discussed and analysed. The aim of the paper is to show the influence of phenomenological ideas on the philosophical conceptions concerning mathematics. We shall start by indicating the attachment of Edmund Husserl to mathematics and by presenting the main points of his philosophy of mathematics. Next, works of two philosophers who attempted to apply Husserl's phenomenological ideas to the philosophy of mathematics, namely Hermann Weyl and Oskar Becker, will be briefly discussed. Lastly, the connections between Husserl's ideas and the philosophy of mathematics of Kurt Gödel will be studied.[1]

Key words: mathematics, philosophy, phenomenology, intuition.

Husserl's Philosophy of Mathematics

Husserl came in a certain sense from mathematics. He began his studies of mathematics at the universities of Leipzig and Berlin with Carl Weierstraß and Leopold Kronecker. In 1881 he moved

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to Vienna where he studied with Leo Königsberger and, in 1883 obtained his doctor's degree on the base of the dissertation *Beiträge zur Variationsrechnung*. Strongly impressed by the lectures of Franz Brentano (1838–1917) on psychology and philosophy which he attended at the University of Vienna, he decided after the doctorate to dedicate his life to philosophy. In 1886 he went to the University of Halle to obtain his *Habilitation* with Carl Stumpf, a former student of Brentano. The *Habilitationsschrift* was entitled Über *den Begriff der Zahl. Psychologische Analysen*. This 64-page work was later expanded into a book (of five times the length), which was one of Husserl's major works: *Philosophie der Arithmetik. Psychologische und logische Untersuchungen* (Husserl, 1891, cf. also Husserl, 1970, 2003).

Working as *Privatdozent* at the University of Halle, Husserl came into contact with mathematicians: Georg Cantor, the founder of set theory and Hermann Grassmann's son, also Hermann. The former, with whom he had long philosophical conversations when they were teaching together in Halle in the 1890s, told him about Bernard Bolzano. In fact, Husserl was perhaps the first philosopher outside Bohemia to be influenced significantly by Bolzano (cf. Grattan-Guinness, 2000). Later, as a professor of the University in Göttingen, Husserl had contact with David Hilbert and as a professor in Freiburg (Breisgau), where he was appointed in 1916, with Ernst Zermelo.

Cantor influenced in a certain sense the earlier works of Husserl though he is quoted only twice in Husserl's *Habilitationsschrift*. Similarly, discussions with Gottlob Frege – the founder of logicism, one of the main trends in the modern philosophy of mathematics – influenced him.[2] Both Cantor and Frege will appear below when we shall describe Husserl's philosophy of arithmetic. Considering the connections of Husserl with mathematics and mathematicians, one can say that his philosophy had, in fact, no visible meaning for the mathematics of his time, however, on the contrary, mathematics strongly influenced his philosophy.

One of the mathematical motives of Husserl's philosophy can be recognized in Weierstraß's program of arithmetization of analysis. Its aim was to found the whole of mathematics on the base of arithmetic,

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[2] For trends in the philosophy of mathematics see for example Bedürftig, Murawski (2015, 2018).

and to define all its concepts in terms of arithmetical ones. Quite a lot of mathematicians of the 19th century initialized and supported this arithmetization, among them Augustin-Louis Cauchy, Bernard Bolzano, Richard Dedekind, Georg Cantor and Carl Weierstraß himself. Husserl's aim was to justify the Weierstraß program by his investigations, philosophically and psychologically. In the Preface to *Philosophie der Arithmetik* he wrote (Husserl, 1891, p. VIII): "Perhaps my efforts should not be wholly worthless, perhaps I have succeeded in preparing the way, at least on some basic points, for the true philosophy of the calculus, that desideratum of centuries".

Aspects of phenomenological methods and basic concepts of phenomenology can already be seen in Husserl's *Habilitationsschrift*. The latter, as well as his book *Philosophie der Arithmetik,* was influenced by Brentano and stamped by his descriptive psychology. Later Husserl moved away from this "psychologism" and criticized the psychological point of view in the philosophy of logic and mathematics – for example in the first volume of his *Logische Untersuchungen* (1900–1901).[3] He was of the opinion that phenomenological data are correctly described by empirical psychology. He changed his mind around 1930 claiming now that there is in fact no direct connection and that psychological analysis cannot be used in phenomenology. This purely philosophically and *a priori* treated phenomenology that should remove psychology as its foundation was developed by Husserl for more than forty years. His aim was to establish philosophy as a strict science and to create the universal foundation of all disciplines.

Mathematics was for Husserl a typical example of an eidetic discipline. According to him, mathematics studies the fundamental objects, like numbers in the arithmetic and forms or similar phenomena in the geometry.[4] Husserl claimed that one can penetrate in a kind of *Wesensschau* to their essences, their *eidos* – as in the case of physical

[3] However some forms of psychologism which he analysed there and tried to reject can be seen not directly in his *Philosophie der Arithmetik*. There are, however, some concepts that appear and are considered both in *Philosophie der Arithmetik* und in *Logische Untersuchungen* but they are treated in a different way. For example in *Philosophie der Arithmetik* an important role is played by the concept of abstraction taken from the psychological point of view. The same term is present in *Logische Untersuchungen* together with a sophisticated theory and many possible variants.

[4] It is worth noting here that Husserl proposed an extension of geometry in the direction called today topology.

objects. He made here no difference. Mathematics, as an eidetic discipline, studies abstract objects in which intentionally is more than we can recognize in our normal cognition and to which we will be phenomenologically led back.

Husserl was not satisfied with the solutions of the program of arithmetization proposed by Dedekind, Cantor and others. His own position, especially in *Philosophie der Arithmetik* was resolutely anti-axiomatic. According to him, one should not found "arithmetic on a sequence of formal definitions, out of which all the theorems of that science could be deduced purely syllogistically" – as he wrote in *Philosophie der Arithmetik* (1891, p. 130, 2003, p. 127). As soon as one comes to the ultimate, elementary concepts, the whole process of defining has to come to an end and one should point to the concrete phenomena from or through which the concepts are abstracted and to show the nature of the abstraction process.

He wrote:

> Today there is a general belief that a rigorous and thoroughgoing development of higher analysis […] excluding all auxiliary concepts borrowed from geometry, would have to emanate from elementary arithmetic alone, in which analysis is grounded. But this elementary arithmetic has, as a matter of fact, its sole foundation in the concept of number; or, more precisely put, it has it in that never-ending series of concepts which mathematicians call "positive whole numbers". […] Therefore, it is with the analysis of the concept of number that any philosophy of mathematics must begin (Husserl, 2003, pp. 310–311).

In *Philosophie der Arithmetik*, Husserl referred, as mentioned above, to Brentano's method of descriptive psychology and understood – similarly to Weierstraß and other mathematicians of that time – natural numbers by empirical counting, what by him is masked by other principles. In the first part of the work, Husserl developed a psychological analysis that started from the everyday concept of a number. The analysis begins with the development, application and appearance of numbers and on this base he tries to explain the psychological origin of numbers. He claims that the fundamental concept of a number cannot be defined:

> […] the difficulty lies in the phenomena, in their correct description, analysis and interpretation. It is only with reference to the phenomena that insight into the essence of the number concept is to be won (Husserl, 1891, p. 142, 2003, p. 136).

These words exhibit Husserl's psychological belief from this period. We find here already the "reference to the phenomena".

Since our intellect and time are bounded, we are able to achieve the comprehension only of a very small part of mathematics. In order to overcome those limits one introduces symbols which accompany and guide our thinking. Almost all we know about arithmetic we know indirectly *via* the intermediation of symbols. This explains why in the second part of *Philosophie der Arithmetik* Husserl considers extensively the symbolic representations.

As indicated above Husserl – being against the axiomatic approach to the characterization of numbers – claimed that the challenge is to find the sources of the number concept, to comprehend the nature of the abstraction process and to describe the concept formation. According to that one should focus on "our grasp of the concept of number" and not on the number as such.

Husserl understands abstraction in the following way: "to abstain from something or abstract from something means simply: not notice this especially". And he explains: abstraction "does not have the effect that its content and its connections disappear from our consciousness" (Husserl 1891, p. 85, 2003, p. 83). It is here psychologically indicated what Husserl later included into his method of phenomenological reduction. That there are contents that are "not especially noticed" – just they make possible the *Colligieren*, the connecting to a new whole.

This *Colligieren*, which leads to "multiplicities", is for Husserl directly connected with the concept of number. This is one of two principles that are fundamental for numbers. The second principle is the principle of "something" underlying everything. "The »something« is no abstract partial content" of any »concrete multiplicity«. For »the concept of something is due to the reflection on the psychic act of conception« (Husserl, 1891, p. 86, 2003, p. 84). Again one can suppose that here – psychological, intentional something – presages the later philosophical eidos.

By such copies of something general, multiplicities are constituted: "A multiplicity is nothing more than: something and something and something etc.; or any one and any one and any one etc.; or briefly: one and one and one etc." (Husserl, 1891, p. 85, 2003, p. 83). In the word "one" Husserl sees the relation of "partial content" with the whole of the multiplicity that is not expressed in "something".

Multiplicity and quantity (*Anzahl*) – and here we are at Husserl's concept of number – can be hardly distinguished. "It is *a priori* apparent that they coincide in their essential content" (Husserl, 1891, p. 89). "Quantity" is the "generic term": the concept of quantity distinguishes the "abstract forms of multiplicities", cancels the "vague indefiniteness" of multiplicities and appends to them the "sharply definite how many" (*loc. cit.*). Multiplicity for Husserl resembles the "something" of number, an indefinable psychological datum (cf. Husserl, 1891, p. 130, 2003, p. 127).

The essential element of the abstraction that leads to the just mentioned concept of quantity is in the concept of "something" (cf. Husserl, 1891, p. 129, 2003, p. 128). It spares – differently as in the case of the set-theoretical concept of a cardinal number – the comparison of "concrete multiplicities", which Husserl explicitly notices. Husserl's concept of quantity comes back to the age-old "definition" of a number by Euclid and corresponds to Cantor's characterization of cardinal numbers stating that it is "a definite aggregate composed of units" (Cantor, 1895, p. 482). We recognize that with Husserl one has to do here with a process which goes like a counting: "One and one and one etc.". He treats numbers as arising and given additively, for example "three" as "one, one and one" (p. 87). The counting process is so explicit and clear in this formulation that it seems that Husserl does not separate quantity and counting. At least in such a way he articulates it in his (very sharp and not consistent) critique of Kant's concept of schemata (cf. Kauferstein, 2006, p. 108 ff.):

> Number is the idea of a universal procedure of imagination getting the concept of quantity an image. However this procedure can only mean counting. But is it not clear that "number" and the idea of "counting" are the same (Husserl, 1891, p. 86, 2003, p. 84)?

This remark is a bit surprising because just at that time Dedekind (1888) and Peano (1889) provided a clear mathematical separation of number and quantity, the grounding of the concept of number by counting. It seems as if the mathematician Husserl did not want to notice this in his psychological, anti-axiomatic attitude. One can briefly characterize Husserl's concept of number by saying that, according to him, numbers are quantities, and quantities are distinguished multiplicities of abstract units.

Just these quantities are for Husserl primary for the concept of number. On the other hand, cardinal numbers as classes of equipollent sets are unfinished and "useless concept formations" (Husserl, 1891, p. 129) which state no number, but only the equality of number or quantity. This "definition" (Husserl himself puts this in quotation marks) is "considerably appreciable" (Husserl, 1891, p. 130 f.) only for "this Wildman" on "that level of mind" for whom the symbolic counting is not available.

In our opinion Husserl misunderstands both Cantor (in favor of him) as well as Frege, and finally also Dedekind. Note that he criticizes only the decided anti-psychologist Frege.

According to Husserl, a mathematician operates not with abstract numbers but with quantities that are always connected with the idea of special sets via multiplicities.

Mathematics itself is for Husserl a formal ontology. Objects investigated by mathematics are formal categories in various forms – and they are themselves not perceivable. Numbers are here an example. Thanks to the ability of categorical abstraction we can free ourselves from the empirical components of judgements and concentrate ourselves on the formal categories. In the eidetic intuition and variation we are able to grasp the possibility, impossibility, necessity and contingency of connections between concepts or between formal categories. The categorical abstraction and the eidetic intuition form the base of the mathematical knowledge.

Comparing Husserl and Frege one sees that for the former a direct experience, i.e., perception, is the ultimate basis for the meaningful analysis of numbers (and other mathematical notions), whereas the latter relies on the certainty given by logic. Husserl wants only to describe our experiences. Frege's logical analysis consists in constructing a notion of number in the ideography. For Husserl, such an approach is artificial or, as he says, "chimerical" (cf. Husserl, 1970, pp. 119–120, 2003, p. 125). He claims that one should analyze concepts as they are given to us.

Weyl's and Becker's Phenomenological Philosophy of Mathematics

The ideas of Husserl found response in papers of the famous German mathematician Hermann Weyl (1885–1955). His interests in philosophy go back to his graduate student days between 1904 and 1908 and his allegiance to it lasted till the early twenties. A few years after the publication of *Das Kontinuum* (1918), Weyl joined the intuitionistic camp of L.E.J. Brouwer and developed his own approach to intuitionism, claiming that philosophy and intuitionism are strongly connected. Later, Weyl changed his views again and legitimated Hilbert's program. All this was connected with his critique of phenomenology. Mancosu and Ryckman (2005, p. 242) claim that "[a]pparently failing to discriminate between the resources available to phenomenology and those of intuitionistic mathematics in accounting for a contentual *Anschauung* capable of grounding the meaning of mathematical statements, Weyl saw the failure of the latter, in the face of Hilbert's finitism, as implicating the failure of the former." Weyl wrote:

> If Hilbert's view prevails over intuitionism, as appears to be the case, *then I see in this a decisive defeat of* the *philosophical attitude of pure phenomenology*, which thus proves to be insufficient for the understanding of creative science even in the area of cognition that is most primal and most readily open to evidence – mathematics [original emphasis] (1967, p. 484).

The influence of Husserl's ideas on Weyl can be seen in the care with which he treated issues like the relationship between intuition and formalization (cf. Weyl, 1918), the connection between his construction postulates and the idea of a pure syntax of relations, the appeal to a *Wesensschau*, etc. In the Preface to the work *Das Kontinuum* Weyl explicitly declares that he agrees with the conceptions that underlie Husserl's *Logische Untersuchungen* with respect to the epistemological side of logic. Answering Husserl's gift of the second edition of *Logische Untersuchungen* to him and his wife, he wrote in a letter to Husserl:

> You have made me and my wife very happy with the last volume of the *Logical Investigations*; and we thank you with admiration for this present. […] Despite all the faults you attribute to the *Logical Investigations* from your present standpoint, I find the conclusive results of this work – which has rendered such an enormous service to the spirit of pure objectivity in epistemology - the

decisive insights on evidence and truth, and the recognition that "intuition" [*Anschauung*] extends beyond sensual intuition, established with great clarity and conciseness (Husserl, 1994, p. 290).

On the other hand Husserl read Weyl's *Das Kontinuum* as well as his *Raum, Zeit, Materie* (1922), and found them close to his views. He stressed and praised Weyl's attempts to develop a philosophy of mathematics on the base of logico-mathematical intuition. Husserl was pleased to have Weyl – who was a prominent mathematician – on his side. In a private correspondence he wrote to Weyl that his works were being read very carefully in Freiburg and had had an important impact on new phenomenological investigations, in particular those of his assistant – Oskar Becker.

Oskar Becker (1889–1964) studied mathematics at Leipzig and wrote his doctoral dissertation in mathematics under Otto Hölder and Karl Rohn in 1914. He then devoted himself to philosophy and wrote his *Habilitationsschrift* on the phenomenological foundations of geometry and relativity under Husserl's supervision in 1923. He admitted that it was Weyl's work that made a phenomenological foundation of geometry possible. Becker became Husserl's assistant in the same year. In 1927, he published his major work *Mathematische Existenz* (1927). The book was strongly influenced by Heidegger's investigations, in particular by his investigations on the facticity of *Dasein*. This led Becker to pose the problem of mathematical existence within the confines of human existence. He wrote: "The factual life of the mankind [...] is the ontical foundation also for the mathematical" (Becker, 1927, p. 636). This standpoint in the philosophy of mathematics led Becker to find the origin of mathematical abstractions in *concrete* aspects of human life. In this way he became critical of Husserl's style of phenomenological analysis. This anthropological current played an important rôle in Becker's analysis of the transfinite. Hence, Becker utilized not only Husserl's phenomenology but also Heideggerian hermeneutics, in particular discussing the infinity of arithmetical counting as "being towards death" (*Sein-zum-Tode*).[5]

[5] Note that being (since 1923) an assistant of Husserl, Becker was attending seminars by Heidegger. This can explain the influence of the latter on Becker's *Mathematische Existenz*. Add that *Mathematische Existenz* and Heidegger's *Sein und Zeit* were published in 1927 in the same issue of *Jahrbuch für Philosophie und Phänomenologische Forschung*.

At the end of his life, Becker re-emphasized the distinction between intuition of the formal and Platonic realm as opposed to the concrete existential realm and developed his own approach to the phenomenology called by him *mantic*. With this word he referred to the fact that there is a divinatory aspect related to any attempt to understand *Natur*. In the light of this mathematics appears as a divinatory science which by means of symbols allows us to go beyond what is accessible. Mantic phenomenology will have to replace the older "eidetic" phenomenology.

Becker's works have not had great influence on later debates in the foundations of mathematics, despite the many interesting analyses included in them, in particular of the existence of mathematical objects.

Talking about Weyl and Becker one should mention also Felix Kaufmann (1895–1949), an Austrian-American philosopher of law. He studied jurisprudence and philosophy in Vienna, and from 1922 till 1938 (when he left for the USA) he was a *Privatdozent* there. He was associated with the Vienna Circle. He wrote on the foundations of mathematics attempting, along with Weyl and Becker, to apply the phenomenology of Husserl to constructive mathematics. His main work here is the book *Das Unendliche in der Mathematik und seine Ausschaltung* (1930).

Gödel's Philosophy of Mathematics *versus* Phenomenology

One of the most eminent logicians and philosophers of mathematics in whom we find Husserl's phenomenological ideas is Kurt Gödel (1906–1978). Let us start by noting that Husserl never referred to Gödel. In fact he was more than 70 years old when Gödel obtained his great results on incompleteness and consistency, and he died a few years later, in 1938. It is claimed, however (cf. Hartimo, 2017), that he knew of Gödel's results. Also Gödel never referred to Husserl in his published works. However his *Nachlass* shows that he knew Husserl's work quite well and appreciated it highly.

Gödel started to study Husserl's works in 1959 and became soon absorbed by them finding the author quite congenial. He owned all Husserl's main works.[6] The underlinings and comments (mostly

[6] He owned among others Logische Untersuchungen (in the edition from 1968), Ideen, Cartesianische Meditationen und Pariser Vorträge, Die Krisis der europäischen Wissenschaften und die transzendentale Phänemenologie.

in Gabelsberger shorthand) in the margin indicate that he studied them carefully. Most of his comments are positive and expand upon Husserl's points, but sometimes he is critical. One should note that Gödel expressed philosophical views on mathematics similar to those of Husserl long before he started to study them (cf. Føllesdal, 1995, p. 428). Views found in Husserl's writings were not radically different from his own. It seems that what impressed him was Husserl's general philosophy which would provide a systematic framework for a number of his own earlier ideas on the foundations of mathematics. Hao Wang (1996, p. 166) writes that "Gödel's own main aim in philosophy was to develop metaphysics – especially, something like the monadology of Leibniz transformed into exact theory – with the help of phenomenology".

Gödel considered both central questions in the philosophy of mathematics: (1) what is the ontological status of mathematical entities, and (2) how do we find out anything about them? Considering the first problem, one should say that Gödel had held realist views on mathematical entities since his student days (cf. Wang, 1974, pp. 8–11) – more exactly since 1921–1922. In *Russell's mathematical logic* he wrote about classes and concepts:

> It seems to me that the assumption of such objects is quite as legitimate as the assumption of physical bodies and there is quite as much reason to believe in their existence. They are in the same sense necessary to obtain a satisfactory systems of mathematics as physical bodies are necessary for a satisfactory theory of our sense perceptions and in both cases it is impossible to interpret the propositions one wants to assert about these entities as propositions about the "data", i.e., in the latter case the actually occurring sense perceptions (Gödel, 1944).

Similar views were expressed by him in his Gibbs lecture (1951/1995) and in the unfinished contribution to the book *The Philosophy of Rudolf Carnap* titled *Is mathematics syntax of language?* (Gödel, 1953/1995). He writes there about concepts and their properties:

> Mathematical propositions, it is true, do not express physical properties of the structures concerned [in physics], but rather properties of the *concepts* in which we describe those structures. But this only shows that the properties of those concepts are something quite as objective and independent of our choice as physical properties of matter. This is not surprising, since concepts are composed of primitive ones, which, as well as their properties, we can create

> as little as the primitive constituents of matter and their properties (Gödel, 1953/1995, p. 9).

It should be stressed that Gödel does not claim here the objective existence of properties, but says only that they are as objective as the physical properties of matter. Compare this with Husserl's claim that abstract objects of mathematics have – like other essences – the same ontological status as physical objects, that they are objective, but not in the straightforward realist sense (cf. Føllesdal, 1995, p. 432, 439).

The comparison of the status of mathematical objects and physical objects one finds also in Supplement to the second edition of Gödel's paper *What is Cantor's Continuum Problem?* where he says (cf. Gödel, 1947/1964, p. 272) that the question of the objective existence of the objects of mathematical intuition is an exact replica of the question of the objective existence of objects of the outer world. Føllesdal (1995, p. 440) notes that "Gödel's use of the phrase »exact replica« brings to mind the analogy Husserl saw between our intuition of essences in *Wesensschau* and of physical objects in perception".

Let us turn now to the second problem, i.e., to the epistemology of mathematics. As indicated above in *Russell's mathematical logic* (1944), Gödel talked about elementary mathematical evidence or mathematical "data" and compared it to sense perception. The notion of mathematical intuition was also discussed by him in the papers (1951/1995) and (1953/1995) quoted above. In 1951 he wrote:

> What is wrong, however, is that the *meaning* of the terms (that is, concepts they denote) is asserted to be something manmade and consisting merely in semantical conventions. The truth, I believe, is that these concepts form an objective reality of their own, which we cannot create or change, but only *perceive* and describe (Gödel, 1951/1995, p. 320).

In 1953 he writes:

> The similarity between mathematical intuition and physical sense is very striking. It is arbitrary to consider "This is red" an immediate datum, but not so to consider the proposition expressing modus ponens or complete induction (or perhaps some simpler propositions from which the latter follows). For the difference, as far as it is relevant here, consists solely in the fact that in the first case a relationship between a concept and a particular object is perceived, while in the second case it is a relationship between concepts (Gödel, 1953/1995, p. 359).

In the Supplement to the second edition of *What is Cantor's Continuum Problem?* he writes:

> But despite their remoteness from sense experience, we do have something like a *perception* of the objects of set theory, as is seen from the fact that the axioms force themselves upon us as being true. I don't see any reason why we should have less confidence in this kind of perception, i.e., in mathematical *intuition*, than in sense perception, which induces us to build up physical theories and to expect that future sense perceptions will agree with them [...] (Gödel, 1947/1964, p. 271).

Gödel did not explain what is the object of mathematical intuition. There are the following possibilities: propositions (cf. Gödel, 1953/1995), concepts (cf. Gödel, 1951/1995), sets and concepts (Gödel, 1947/1964), or all three. Recall that Husserl distinguished two kinds of intuition: perception (where physical objects are intuited) and eidetic intuition (where the object is an eidetic entity or a "something" according to his *Philosophie der Arithmetik*) and claimed that the latter is more basic. It is not clear whether Gödel shared his views in this respect.

It is worth quoting still one passage from the second edition of (Gödel, 1947/1964) where he wrote:

> That something besides the sensations actually is immediately given follows (independently of mathematics) from the fact that even our ideas referring to physical objects contain constituents qualitatively different from sensations or mere combinations of sensations, e.g., the idea of object itself. [...] Evidently the "given" underlying mathematics is closely related to the abstract elements contained in our empirical ideas. It by no means follows, however, that the data of this second kind, because they cannot be associated with actions of certain things upon our sense organs, are something purely subjective, as Kant asserted. Rather they, too, may represent an aspect of objective reality, but, as opposed to the sensations, their presence in us may be due to another kind of relationship between ourselves and reality (p. 271).

Føllesdal (1995, p. 442) suggests that Gödel's point in this passage is that "what is given in our experience is not just physical objects, but also various abstract features that are instantiated by these objects".

Mathematical intuition cannot guarantee us certainty of our knowledge. In fact neither perception nor categorical intuition are infallible sources of evidence. Gödel writes about four different methods one can use to get insight into mathematical reality:

- elementary consequences,
- success, i.e., fruitfulness in consequences,
- clarification,
- systematicity.

The first one is involved in the situation when recondite axioms have elementary consequences, e.g., axioms concerning great transfinite numbers can have consequences in the arithmetic of natural numbers. Clarification refers to situations when a discussed hypothesis cannot be solved generally, but it is solvable with the help of some new axioms (compare the problem of the continuum hypothesis and the axiom of constructibility). The last, systematicity, refers to the method of arranging the axioms in a systematic manner what enables us to discover new ones.

The last method (that recalls Husserl's "reflective equilibrium" approach to justification) was mentioned by Gödel in the manuscript *The modern development of the foundations of mathematics in the light of philosophy* (1961/1995). Gödel described there in philosophical terms the development of the study of the foundations of mathematics in the 20th century and fitted it into a general scheme of possible philosophical *Weltanschauungen*. Among others he discussed also Husserl's philosophy, finding in it the method for the clarification of meaning of mathematical concepts.[7] He wrote there:

> […] it turns out that in the systematic establishment of the axioms of mathematics, new axioms, which do not follow by formal logic from those previously established, again and again become evident. It is not at all excluded by the negative results mentioned earlier that nevertheless every clearly posed mathematical yes-or-no question is solvable in this way. For it is just this becoming evident of more and more new axioms on the basis of the meaning of the primitive notions that a machine cannot imitate (Gödel, 1961/1995, p. 385).

Gödel refers here to his famous incompleteness results from (1931). They state that (1) every consistent theory containing the arithmetic of natural numbers contains undecidable propositions and that (2) no such theory can prove its own consistency. Those results showed that neither Hilbert's program of justification of the classical mathematics by means of finitary methods, nor Carnap's syntactical

[7] This is the only place in which Gödel mentions explicitly Husserl and his philosophy.

program reducing mathematics to its syntax, can be realized. Hence the rôle of mathematical intuition, which can help us to find out deeper meaning and properties of mathematical concepts that are not included in definitions given by axioms. Gödel says in (1961/1995) that there "exists today the beginning of a science which claims to possess a systematic method for such clarification of meaning, and that is the philosophy founded by Husserl". And he continues:

> Here clarification of meaning consists in concentrating more intensely on the concepts in questions by directing our attention in a certain way, namely, onto our own acts in the use of those concepts, onto our own powers in carrying out those acts, etc. In so doing, one must keep clearly in mind that this philosophy is not a science in the same sense as the other sciences. Rather it is [or in any case should be] a procedure or technique that should produce in us a new state of consciousness in which we describe in detail the basic concepts we use in our thought, or grasp other, hitherto unknown, basic concepts (Gödel, 1961/1995, p. 383).

This path of Gödel's from the incompleteness results to philosophy is not surprising. In a sense, the incompleteness theorems support and are supported by phenomenological views. They support philosophy because they suggest that an intuition of mathematical essences or a grasp of abstract concepts that cannot be understood on the basis of axioms alone is required in order to solve certain problems and to obtain consistency proofs for formal theories. On the other hand, they are supported by philosophy because the latter gives mathematical essences their due. Gödel claimed that it is necessary to ascend to stronger, more abstract principles and axioms to be able to solve problems from the lower levels (for example to set theoretic principles to solve number theoretic problems). This idea was strongly supported by the results of Paris, Harrington and Kirby which provided examples of genuine mathematical statements that refer only to natural numbers, that are undecidable in number theory, but that can be solved by using infinite sets of natural numbers.[8]

In the paper *The modern development of the foundations of mathematics in the light of philosophy* (1961/1995) Gödel says also that it is not excluded that every clearly formulated mathematical yes-or-no question can be solved through cultivating our knowledge of abstract concepts,

[8] Cf. Paris, Harrigton (1977) and Kirby, Paris (1982). See also Murawski (1984).

through developing our intuition of essences. In fact in this way more and more new axioms become evident on the basis of the meaning of the primitive concepts that a machine, i.e., a formal procedure, cannot emulate.

It seems that Gödel settled on Husserl's philosophy because according to it we are directed toward and have access to essences in our experience – and this is a support for platonism which was Gödel's favorite conception in the philosophy of mathematics.

Conclusion

Husserl's post-psychologistic, transcendental view of mathematics is still a live option in the philosophy of mathematics. As Tieszen writes it is "compatible with the post-Fregean, post-Hilbertian and post Gödelian situation in the foundations of mathematics" (cf. Tieszen, 1994, p. 335). The phenomenological approach to the philosophy of mathematics is still being developed by various authors. The starting point for their considerations are, however, not directly Husserl's works but rather Gödel's considerations. Let us mention here, for example, P. Benacerraf, Ch. Chihara, P. Maddy, M. Steiner, Ch. Parsons and R. Tieszen. They are commenting on Gödel's works concentrating in particular on the problem of mathematical intuition – cf., for example, Maddy (1980), Parsons (1980) or Tieszen (1988).

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