STUDIA SEMIOTYCZNE (SEMIOTIC STUDIES), 39(2)
ISSN 0137-6608, e-ISSN 2544-073X
DOI: 10.26333/sts.xxxix2.04

Article

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TWO WAYS OF USING SIGNS. A LOGICAL CONCEPTION OF MEANING AS A WAY OF USING EXPRESSIONS

SUMMARY: In the paper, I outline the general, logical conception of meaning which is based on understanding it as a creation determined trough the way of using linguistic expressions. The concept of meaning is based on ideas deriving from L. Wittgenstein (1953) and independently from K. Ajdukiewicz (1931, 1978), and it is connected with the so-called *functional approach to analysis of natural language* – the notion introduced by Jerzy Pelc (1971, 1979). This is marked by taking into consideration the manner of use expressions. Following Pelc, we distinguish two understandings of this statement: in the first of them, the manner of *use* concerns only expression-*tokens* (material sings); in the other – the manner of *Use* (usage) characterizes the meaning of expression-*types* (classes of expression-*tokens*). This manner is somehow built into the meaning. The relation of *use*, concerning all the relations of physical object-based reference of expression-*tokens* made by users of language, will be a primitive notion of the theory outlined here. The relation *Use,* usage, is, on the other hand, a relation defined by means of the relation of *use* and applied by users of language to expression-*types*. The meaning of the expression-*type e* is defined as the equivalence class of the relation *possessing the same manner of Use* (usage) of *types* determined by *type e*. It may intuitively be understood as a common property of all expression-*types* possessing the same manner of *Use* (usage) as the expression *e*. In accordance with the well-known differentiations: *Sinn-Bedeutung* of G. Frege and *intension-extension* of R. Carnap, the notion of denotation differs from that of meaning and, in the paper, is defined by means of the relation *referring of expression-type* to objects of reality described by the given language.

KEYWORDS: token-type distinction, Pelc's functional approach to logical semiotics, using expression token/type, denotation, meaning.

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1. Introduction

Language, as a system of signs, is not something empirically given. It is a theoretical creation with given properties; it has a conventional character. Its signs and expressions perform various functions studied by researchers. These functions are different when its signs and expressions are understood either *in concreto* (as sign/expression-*tokens,* material entities) or *in abstracto* (as sign/expression-*types, abstract entities*). The former perform mainly representational and communicative-cognitive functions, whereas the latter – semantic, meaning-related functions.

The main goals of this paper are: 1) presenting the mutual interactions of these functions of signs and linguistic expressions of a dual ontological nature and 2) providing logical explication of their semantic-pragmatic properties, by taking into account the so-called *functional approach to analysis within the logical theory of language*, i.e. within logical semiotics and, in connection with this approach, 3) outlining the general, logical, theoretical, conception of meaning as a semantic-pragmatic formal-logical theory. Its task is to explain and explicate the general notion of meaning and also to characterize principal relations between this and related notions, such as reference and denotation.

The general theory of meaning attempts to answer the question: < What are meaning, reference and denotation at all?>. It is a part of general semantics and general pragmatics.

Although the main aim of the present work is to outline the foundations of a general axiomatic formal-logical theory of meaning of expressions of any language, it will take into consideration, to a certain degree, the *functional approach* to logical analysis of natural language.

The concept of the functional approach to the logical semiotics of natural language was introdu- ced by Jerzy Pelc (1971, 1979). It is based on two ways of using linguistic signs and expressions. One of them is for sign/expression-*tokens* and the other – for sign/expression-*types*.

In Section 2, we will start our considerations by recalling the *token-type* distinction of linguistic signs introduced into semiotics by Ch.S. Peirce (1906, p.505 and 1977; see also Peirce 1893-1911, p. 480 and Peirce 1931-1935). Then, in Section 3, according to Pelc's framework, we provide an explanation of what the functional approach to the logical semiotics of natural language is. In the subsequent parts of the paper, we outline the general logical, theoretical conception of the meaning based on the intuition that meaning is determined by the way of using linguistic expressions.

2. The Dual Ontological Nature of Linguistic Sings and Expressions

In a semiotic analysis and a linguistic one, there are used both *types* and *tokens* of signs, often without paying due attention: when *types* are spoken about, and when *tokens* are. This is in connection with the problems which are continuously discussed in philosophy of language: What is the *type* of a sign? What is the *token* (specimen) of a sign? What are mutual relations between the *type* and the *token*?

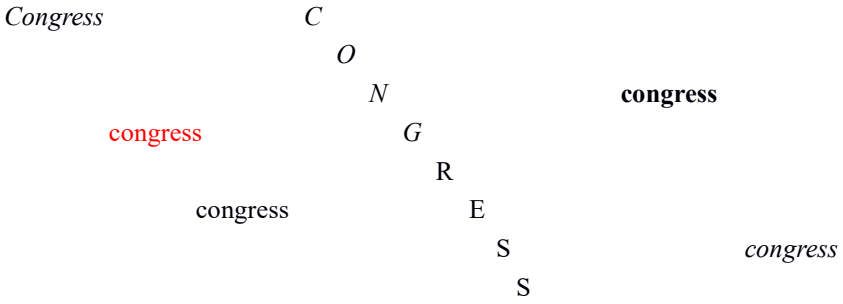
Let us consider, for instance, the following arithmetical equation:

$$2 + 2 = 2 \times 2$$

and let us ask: How many signs/words are there? The answer is indeed ambiguous. In one sense, there are four, yet in another – seven. This depends, obviously, on whether we mean sign/word-*types* or sign/word-*tokens.*

It is most often assumed that word-*tokens*, *tokens*, specimens are material, physical objects – concretes – localized in time and space. They can be formed from ball pen ink, pixels of light, electronic dots or sections (or their lack) on a computer screen.

The basic difference between *types* of words and their *tokens* is marked in that, the same *type* can be exemplified by different *tokens*. For instance, the word-*type* called "congress" can be exemplified by the following different, but from a pragmatic point of view indiscernible, *identifiable* word-*tokens*:



Types are most often understood as some abstract sets, classes of *identifiable tokens*, whose only mutual feature is structural similarity (spelling or phonographic) – thus not necessarily physical similarity: the same shape or the same sound. They are abstract entities, abstract ontological beings while their *tokens* are concrete entities, material ontological beings.

3. Functional Approach to Logical Analysis of Expressions of Natural Language. Two Ways of Using Its Expressions

What is the functional approach to analysis within the logical semiotics of natural language? Jerzy Pelc gave (in 1979, the beginning of Chapter II) the following concise answer: It consists in taking into account *how the analyzed expression was used.* Consequently, he distinguished two ways of using linguistic verbal signs and expressions.

In the first of them, the *way, manner of use*, takes place only in given conditionings, in a determined language-situational context and concerns only expression-*tokens*. It is then in concrete acts of communicating in given linguistic-situational context with reference of these *tokens* to extralanguage objects.

In the other – the *way*, *manner of Use* (*usage*) characterizes the *meaning* of an expression. This manner is somehow built into its meaning. In this case, the expression can be treated as isolated, static, torn from the context, e.g. as an entry in a dictionary. This concerns any expression-*type,* a class of its concrete occurrences, a distributive class of its *identifiable* expression-*tokens.*

When Jerzy Pelc writes about the functional approach to natural language analysis, he means mainly the first way of understanding the use of verbal linguistic signs and language expressions – the manner of *use.* However, he emphasizes that each such *use* of any linguistic expression (any expression-*token*) is compatible with the manner of *Use* (*usage*) of its expression-*type,* in the second sense of the way of using linguistic expressions, and therefore with its *meaning* adopted in a given language by its competent users.

Language *tokens* (concretes) alone do not have either a meaning or denotation (they can only be *used* in the meaning defined by their *types*). Different, identifiable *tokens* of an expression-*type* with an unambiguous meaning (fixed way of *Use*) can be *used* (in the first sense) either to represent a given object, or in concrete acts of communication in given linguistic-situational contexts, with reference to only one object, a widely conceived object, or to more than one object, yet of the same kind. Thus, two different *tokens e* and *e'* of the same expression-*type* $\underline{e}$ can have for their user *u* or users *u*, *u'* the same object reference *o* (see Fig. 1) or, e.g. in an act of communication – the same or two different object references *o* and *o'* (see Fig. 2), when the sender *s*, *uses* a token *e* with reference to the object *o*, while the recipient *r interprets*, *uses* a token *e'* identifiable with *e* in compliance or in discordance with the sender's intention, as the object *o* or another object *o*'. Compliance produces understanding, while discordance produces misunderstanding.

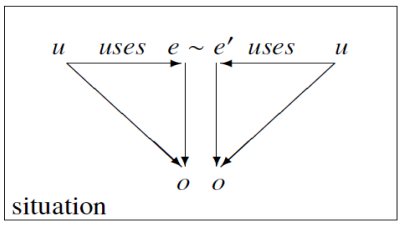


Fig. 1

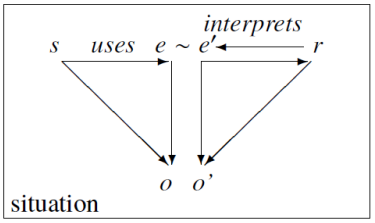


Fig.2

For example, two single *tokens* of the word-*type* 'congress', having a fixed manner of *Use* (the fixed meaning) in English, can be *used* in a similar linguistic-situational context either with reference to the same congress, e.g. the last semiotic congress, or with reference to two different congresses, e.g. in a situation of comparing organizations of the last and the previous semiotic congress.

Similarly, the same *token* of a sentence-type – like "This government must fall" – placed on a banner in different temporal and spatial locations, in different situations, can have different object reference, different demands.

Introduced by Pelc in (1971, 1979) both of the above-mentioned different ways of using linguistic verbal signs and expressions of language: the manner of *use* and the manner of *Use* (usage) are taken into account to formalize the semantic-pragmatic concepts of the general, logical conception of meaning outlined in the paper.

The relation *use*, concerning all the relations of physical object-based reference of expression- *tokens* made by users of language, will be a primitive notion of the proposed theory. This relation is understood as a set of all such physical relations. The relation *Use* is, on the other hand, a relation defined by means of the relation *use* and applied by users of the language to expression-*types*. The difference between these two relations is explained by the fact that two persons can *Use* the same expression *type*, e.g. the word *type* "congress", by means of its two different *tokens*, that is by *using* its two different *tokens*; in our example – in a talk, two persons can *Use* the same expression-*type* "congress", talking by means of its two spoken *tokens* about one or two different congresses.

4. An Outline of the General Logical Conception of Meaning

4.1. Preliminaries

The word "meaning" possesses many different meanings. This word, lacking precision as regards its meaning, requires a logical explication. Despite the fact that the word "meaning" is included in the key terms of logical semiotics and philosophy of language, it does not really have a fixed meaning in these disciplines. There are many different conceptions of meaning and many hypotheses

concerning the nature of meaning. None of the known theories of meaning has come to be commonly accepted.

An attempt at a logical explication of the notion 'meaning' and some other notions of current general semantics and pragmatics is the general, axiomatic, formal-logical theory of meaning presented in my paper "Meaning and Interpretation" (2007a,b) and in my book "Logic-Language-Ontology" (2022, Chapters 5 and 6).

In the theory, which is a general logical conception of meaning, the understanding of the meaning of expressions as the manner of their *Use* by users of language is taken into account. Such an approach refers to the conception of meaning originated from Wittgenstein (1953) and, independently, from Ajdukiewicz (1931, 1978), who, in his directive theory of meaning, establishes a relationship between the meaning of an expression and the way in which we use that expression in language; this relationship can be formulated in meaning directives.

The basic notions of the presented theory of meaning are defined by means of well-formed expression-*types* (wfe-*types*), but their definitions involve such a primitive notion of the theory as the relation *use* for well-formed expression-*tokens* (wfe-*tokens*) and a notion of the relation *Use* (*usage*) for wfe-*types* defined with it. So, the starting point here is Pelc's *use*-*usage* (*use*-*Use*) distinction.

A language *L* is formalized dually: both as a language of *tokens* (at the *token* level) and a language of *types* (at the *type* level), according to the *token-type* distinction of a sign by Peirce.

A formalization of a theory of language syntax is given in my books (1991, 2022, Chapters 1 and 2).

The most important syntactic notion of a *well-formed expression* (a wfe) is defined separately on the *token* level and on the *type* level.

The set of all wfe-*tokens* of *L* is denoted by *S,* and the set of its wfe-*types* which are not syntactic nonsenses or self-contradictions (oxymorons) – by S*.

Every wfe-*type* of S* is a class of identifiable wfe-*tokens* of *S.*

The wfe-*tokens* of *S* are marked with letters *e, e'* and wfe-*types* of *S** - with letters *e*, *e'*.

Two-level formalization of syntax of language *L* allows us to outline the semantic-pragmatic theory of meaning – the general meaning theory ***MT***.

4.2. Basic Axioms, Definitions and Theorems of *MT*

The primitive notions of the theory ***MT*** are:

- the set *User* of all competent users of a given language *L*,
- the set *Ont* of all extralinguistic objects described by *L*, and
- the ternary relation *use* of using by users of *L* the wfe-*tokens* with reference to *extralinguistic* objects of *Ont.*

These primitive notions of the theory ***MT*** are understood very broadly. They satisfy only the following axioms:

$$\text{AXIOM (sets: } \mathit{User} \text{ and } \mathit{Ont}\text{):} \qquad \mathit{User} \neq \emptyset \ \wedge \ \mathit{Ont} \neq \emptyset.$$
$$\text{AXIOM } (\mathit{use})\text{: } \mathit{use} \text{ is a partial function of}$$
$$\mathit{User} \times S \longrightarrow \mathit{Ont},$$
$$\mathrm{Dom}_1(\mathit{use}) = \mathit{User} \text{ and } \mathrm{Dom}_2\ (\mathit{use}) \subset S.$$

Axiomatically, we only assume that the sets *User* and *Ont* are non-empty sets. The set *User* of competent users of the language *L* can be composed not only of current, but also former or future users of this language. A competent user of a language is only one who accurately applies its grammatical and semantic rules. We do not assume anything about the nature of the objects of the set *Ont*. They may belong to various ontological categories. They can be not only material objects but also certain classes of such objects, properties, relations, states of affairs, fictional or abstract creations described by the language *L* as well. Therefore, an object of the set *Ont* may be anything that one talks about, thinks about, etc., by means of expressions of the language *L*.

We understand the relation *use* also in a very broad sense: as an operation of producing, calling, using, exposing or interpreting wfe-*tokens* in order to refer them to corresponding objects of *Ont*. We can also call the operation *use* a function of object reference of wfe-*tokens* by users of the language *L*. Formally, in accordance with the second axiom, it is a partial function defined on the set of pairs: (*user, wfe-token*), and its values are objects of the set *Ont.* It follows from the axiom that every competent user of *L uses* at least one wfe-*token* of the set *S* to refer to an object of *Ont*. This object is called the *referent of the wfe-token assigned by its user.* Not every *wfe-token* must have a referent (nonsense expression-*tokens* have no referent in language). For example, the following absurd sentence-*token* written below:

A completely blind man sees

has no referent.

If $u \in User$, $e \in S$ and $o \in Ont$, the expression:

$$use(u, e) = o$$

will be read: *the user u uses the wfe-token e to refer to the object o.*

DEFINITION (possessing of object reference)

$$e \; has \; object \; reference \text{ iff } e \in S \wedge \exists u \in User \exists o \in Ont(use(u, e) = o).$$

In accordance with the above Definition: an object reference has only such wfe-*token e* that is *used* by some user *u* of *L* to refer to an extra-linguistic object *o*.

For example: the object reference has the sentence-*token*

Robert Lewandowski is a football player

when I am the user who *uses* this sentence with object reference to the referent: the state of being a football player by Robert Lewandowski.

Let us also note, for example, that the object reference has the subject-name in the sentence-*token*

A dog likes its master

and its referent is the master's dog, while the referent of the subject term in the sentence-*token*

Dog is a mammal

is the entire class of all dogs.

The referents of the entire sentences above are the relevant states of affairs.

We mentioned in Sec. 3 that two different expression-*tokens* can have the same object references (see Fig.1) when they have the same way of *use*. It follows from the following definition:

DEFINITION (*possessing the same manner of use*)

$$e \approx e' \text{ iff } \exists o \in Ont[\exists u \in User(use(u, e) = o) \wedge \exists u \in User(use(u, e') = o)],$$

i.e. wfe-*tokens e* and *e'* have the same manner of *use* if and only if they have the same referent.

For example, both the proper name I *used* for Robert Lewandowski and the one a commentator *used* in a football match with Robert Lewandowski refer to the same person: Robert Lewandowski, the football player.

The relation *Use* for wfe-*types* of *L* satisfies the following

AXIOM (*Use*):

$$\emptyset \neq Use \subseteq User \times \underline{S}^*,$$

so, *Use* is a non-empty relation defined on pairs: *user, wfe-type*, and it is defined by means of the operation *use* for wfe-*tokens* as follows:

DEFINITION (*Using types*):

$$u \ Use \ \underline{e} \text{ iff } \exists e \in \underline{e} \ \exists o \in Ont \ (use(u, e) = o).$$

In accordance with this definition, a user *u Uses* the wfe-*type* $\underline{e}$ if and only if the user *u uses* a wfe-*token* of $\underline{e}$ to refer to some referent.

For instance: I *Use* the word "laptop" when I am just saying

this is my laptop

and I am pointing at my laptop.

Immediately from the above axioms and Definition (possessing of reference), there follows

COROLLARY 1:

a. $\forall u \in User \ \exists e \in S \ \exists o \in Ont \ (use(u, e) = o)$,
b. $\forall u \in User \ \exists \ \underline{e} \in \underline{S}^* \ (u \ Use \ \underline{e})$,
c. $\neg u \ Use \ \underline{e}$ iff $\forall e \in \underline{e} \ \forall o \in Ont \ (\neg use(u, e) = o)$.
d. If e has object reference, than $\exists u \in User \exists \underline{e} \in \underline{S}^* \ (e \in \underline{e} \wedge u \ Use \ \underline{e})$.

So,

a. Every user *use*s a wfe-*token* to refer to some object.
b. Each user *Uses* some wfe-*type*.
c. The user does not *Use* the wfe-*type* if and only if he does not *use* any of its wfe-*tokens to refer to any object.*
d. If a wfe-*token* has object reference, then there is a user who *Uses* a wfe-*type* of this wfe-*token.*

Possessing referents, and thus object reference, distinguishes wfe-*tokens* from their linguistic *types* that may only have meaning and denotation but lack a direct connection to objects in the world. In contrast, denotation involves a broader

relationship, encapsulating not only the specific instance a *token* refers to, but also the set of all possible objects a wfe-*type* can denote within a language.

More precisely, while object reference is always tied to a particular usage event involving a specific user and *token,* denotation abstracts away from these particulars to capture the general range of objects that an expression-*type* could be used to refer to across all contexts. This distinction is crucial for understanding how language functions both in individual acts of communication and in terms of its broader semantic structure.

To clarify, object reference is inherently context-dependent and involves a direct act by a user, whereas denotation provides a context-independent mapping between expression-*types* and the set of all objects they could possibly refer to. This separation highlights the dynamic interplay between how language is used in actual communicative situations and how its elements are systematically organized in terms of meaning.

This distinction between object reference and denotation is reflected in the formal structure of the language. Object reference links a wfe-*token* to a particular object through an act of *use* by an individual, grounding language in real-world communicative events. Denotation, on the other hand, catalogs all the potential objects that a wfe-*type* could apply to, regardless of whether those objects are currently being referenced. Understanding this dual layer is essential for analyzing both the pragmatic and semantic dimensions of linguistic expressions.

The notion of *denotation* in ***MT*** is formally defined by means of the relation *referring Ref.* It is a binary relation between wfe-*types* and objects of the set *Ont.* Formally it is a set of the ordered pairs: (wfe- *type*, object) of the Cartesian product of the set $\underline{S}^*$ of wfe-*types* and the set *Ont,* i.e.

$$Ref \subseteq \underline{S}^* \times Ont,$$

and its definition is:

DEFINITION (*referring*):

$$\underline{e}\ Ref\ o \text{ iff } \exists u \in User\ \exists e \in \underline{e}\ (use(u, e) = o).$$

The wfe-*type* $\underline{e}$ refers to the object *o* iff there exists a user of the language *L* *using* some *token* of the expression $\underline{e}$ to refer to the object *o*. Thus, the wfe-*type* $\underline{e}$ does not refer to the object *o* iff no competent user of the language *L* uses any *token* of the expression $\underline{e}$ to refer to the object *o*. For example, the term "pen" refers to a pen on my desk, but does not refer to any computer.

Every object to which $\underline{e}$ refers is called a *denotatum* of the expression-*type* $\underline{e}$. The set of all denotata of $\underline{e}$ is denoted by $\delta(\underline{e})$ and called the *denotation* (*extension*) of the expression-*type* $\underline{e}$. Thus,

DEFINITION (*denotation*):

$$\delta(\underline{e}) = \{o \in Ont\colon \underline{e}\ Ref\ o\}.$$

By applying the above definitions we can state that every denotatum of the wfe-*type* $\underline{e}$ is a referent of some of its *token.*

So, we obtain the following:

THEOREM:

The denotation $\delta(\underline{e})$ of the wfe-*type* $\underline{e}$
is the set of all references of wfe-*tokens* of $\underline{e}$.

For example: Denotation of the term “congress” is the set of all referents of its *tokens*, i.e. the set of all congresses, and denotation of the sentence “It's raining” is the set of all current situations of raining at some moments, while the denotation of the sentence “Rain rains” is the set composed with the abstract state that rain has the property of raining.

It is easy to deduce in ***MT*** that the following conclusion holds:

COROLLARY 2:

$$\forall u{\in}User\ \forall e,e'{\in}\underline{e}\ \forall o,o'{\in}Ont[(use(u,\ e) = o) \wedge use(u,\ e') = o') \Rightarrow o,o'{\in}\delta(\underline{e})].$$

It says that if two expression-*tokens* of the same wfe-*type* have two referents, respectively, then these objects belong to the denotation of the expression-*type*. More exactly: for any user *u*, for any couple of *tokens* *e*, *e'* of the wfe-*type* $\underline{e}$ and for any objects *o*, *o*' if the user *u* *uses* the expression *e* to refer to the object *o* and the expression *e'* to refer to the object *o'*, then referents *o*, *o'* are equal if *e* and *e'* possess the same way of *use* (e.g. if $\underline{e}$ is a proper name) or they are different but belong to the denotation of the expression $\underline{e}$ (e.g. if $\underline{e}$ is a general term).

If we accept the convention adopted by linguists that if $\delta(\underline{e}) = \{o\}$ and $\underline{e}$ is a proper name, then its denotation equals *o*. So, for non-nominalists, denotations of any proper names are individual objects, i.e. their denotata, while the denotations of predicate-names are sets of all their denotata.

Let us observe that from the above definitions it follows that the denotation of the expression-*type* $\underline{e}$ is a nonempty set of all its denotata if and only if a user of the language *L* *Uses* the expression $\underline{e}$. So, we can formulate

THEOREM:

$$\text{a. } \exists u \in \mathit{User}\ (u\ \mathit{Use}\ \underline{e}) \text{ iff } \delta(\underline{e}) \neq \emptyset;$$
$$\text{b. } \delta(\underline{e}) = \emptyset \text{ iff } \forall u \in \mathit{User}\ (\neg u\ \mathit{Use}\ \underline{e});$$
$$\text{c. If } \forall u \in \mathit{User}\ (u\ \mathit{Use}\ \underline{e}) \text{ then } \delta(\underline{e}) \neq \emptyset.$$

In accordance with part b. of the theorem, a name, e.g. the name "elf" has an empty denotation iff no user of *L* in which this name functions, applies any of its occurrences (*token*) to any extra-language objects, or in other words, if there is no user of *L* who would refer by any occurrences (*token*) of the word (e.g. the word "elf") to anything. If, then, all users of the language *L* assume that objects described by this language, objects of the set *Ont*, cannot be fictional beings, this name is treated as empty since it does not have denotata (it does not have designates either, that is referents existing in reality). When somebody uses the fairy-tale name "elf" and does not follow the nominalistic view, and includes fictional beings in the set *Ont*, then the denotation of the name "elf" is a non-empty set to this person and the name functions in this person's language as a general nonempty term (there are a great number of elves in the world of fables). So, in this approach to semantics, the denotata of the so-called "empty names', that are not self-contradictory names, can be fictional or intentional non-physical objects.

So far, we have used two ways of using expressions of the language *L* to define one of the basic semantic-pragmatic notions of the theory ***MT***, i.e. the notion of the *denotation* of an expression-*type* of this language. These two ways of using expressions of *L* are used to define the *meaning* of an expression-*type* – the fundamental concept of general semantics and the theory ***MT***.

The notion 'meaning' differs from the notion 'denotation'. We own the differentiation *meaning-denotation* (*Sinn und Bedeutung*) mainly to Gottlob Frege (1997). The notion of meaning as *intension* in opposition to the notion of denotation as *extension* was introduced by Rudolf Carnap (1947).

In the proposed theory of meaning ***MT*** to define the notion of *meaning* (*intension*) we are taking into account the manner of *Use* (*usage*) which concerns only wfe-*types*, yet, which employs the way of *use* of wfe-*tokens.* The fundamental notion of ***MT*** – the notion of meaning, is defined by means of the *relation* ≅ *possessing the same manner of Use* of the wfe-*types*:

DEFINITION (possessing the same manner of *Use* of *types*):

$$\underline{e} \cong \underline{e}' \text{ iff } \forall u\ \mathit{User}\ [(u\ \mathit{Use}\ \underline{e} \Leftrightarrow u\ \mathit{Use}\ \underline{e}') \wedge$$
$$\wedge\ \forall o \in \mathit{Ont}\ (\exists e \in \underline{e}\ (\mathit{use}(u, e)=o) \Leftrightarrow \exists e' \in \underline{e}'\ (\mathit{use}(u, e')=o))].$$

The above definition states that two wfe-*types* $\underline{e}$, $\underline{e}'$ have the same manner of *Use* if and only if any user of the given language *L Uses* one of them if and only

if he/she also *Uses* the second of them, and every object is a referent of some *token* of the first wfe-*type* $\underline{e}$ (*used* by the user) if and only if it is a referent of some *token* of the second wfe-*type* $\underline{e'}$ (*used* by the user).

For example: the terms "gift" and "present" have the same manner of *Use* because if any user *Uses* both of them, he/she *uses* a *token* of the term "gift" to refer to any object which is a gift iff he/she *uses* some *token* of the term 'present' to refer to the same object, to the same gift.

The relationship between two different relations of possessing the same manner of use is described by

THEOREM:

$$\exists u \in User(u \ Use \ \underline{e}) \wedge \underline{e} \cong \underline{e'} \Rightarrow \exists e \in \underline{e} \exists e' \in \underline{e'}(e \approx e').$$

It states that if two *Used* wfe-*types* $\underline{e}$ and $\underline{e'}$ have the same manner of *Use* (in the second sense), then there exist *tokens* e and e' of these wfe-*types* $\underline{e}$ and $\underline{e'}$, respectively, that have also the same manner of *use* (in the first sense), i.e. that have the same referents.

From the definition of the relation ≅ it follows that it is a reflexive, symmetric and transitive relation, and we get

THEOREM: *The relation* ≅ *possessing the same manner of Use is an equivalence relation in the set* $\underline{S^*}$ of wfe-*types* of the language *L.*

The notion of *meaning* of any wfe-*type* $\underline{e}$ of the set $\underline{S^*}$ is defined in the following way:

DEFINITION (*meaning*):

$$\mu(\underline{e}) = [\ \underline{e}\ ]_{\cong}.$$

The *meaning* (*intension*) $\mu(\underline{e})$ of the wfe-*type* $\underline{e}$ is the equivalence class of the relation ≅ possessing the same manner of *Use* of *types,* determined by wfe-*type* $\underline{e}$.

So, the meaning $\mu(\underline{e})$ of the wfe-*type* $\underline{e}$ is determined by the way (manner) of *Use* of expression *types* of the language *L* and it may intuitively be understood as a common property of all wfe-*types* possessing the same manner of *Use* as the expression-*type* $\underline{e}$.

From Definition (*meaning*) and Definition (*possessing the same manner of Use of types*) the following corollary follows directly

COROLLARY 3:

$$\mu(\underline{e}) = \mu(\underline{e}') \text{ iff } \underline{e} \cong \underline{e}',$$

So, expressions that have the same meaning have the same manner of *Use*.

DEFINITION (*synonymous*):

$$\underline{e} \text{ and } \underline{e}' \textit{ are synonymous } \text{iff } \mu(\underline{e}) = \mu(\underline{e}').$$

So, *two wfe-types are synonymous iff they have the same meaning.*

For example: the predicate-names "teenager" and "a person who is between 13 and 19 years old" are synonymous in English. As well as "elevator" and "lift".

According to Definition (*synonymous*), Definition (*meaning*) and Corollary 1 we conclude:

COROLLARY 4:

Meaning of the wfe-type $\underline{e}$
is the equivalence class of all expressions synonymous with $\underline{e}$.

The basic relationships between the notions of meaning (*intension*) and denotation (*extension*), already known to Frege, in the theory ***MT*** establish the following

THEOREM:

$$\mu(\underline{e}) = \mu(\underline{e}') \rightarrow \delta(\underline{e}) = \delta(\underline{e}')$$

So, *if two expresion-types have the same meaning* (*intensions*), *then they have the same denotation* (*extensions*).

Proof (by *a reductio ad absurdam*).

Let us assume that

(1) $\mu(\underline{e}) = \mu(\underline{e}')$ and

(2) $\delta(\underline{e}) \neq \delta(\underline{e}')$.

From (1) and Corollary 3 we have

(3) $\underline{e} \cong \underline{e'}$.

Thus, from Definition (*possessing the same manner of Use of types*) we get

(4) $\forall u \in User\,[(u\ Use\ \underline{e} \Longleftrightarrow u\ Use\ \underline{e'}) \wedge$

$$\wedge\ \forall o \in Ont\ (\exists e \in \underline{e}\ (use(u, e) = o) \Longleftrightarrow \exists e' \in \underline{e'}\ (use(u, e')) = o)].$$

From the assumption (2) we state that there exists some object $o_1 \in Ont$ such that

(a) $o_1 \in \delta(\underline{e})$ and $o_1 \notin \delta\ (\underline{e'})$

or

(b) $o_1 \in \delta(\underline{e'})$ and $o_1 \notin \delta\ (\underline{e})$.

We have to justify that in both cases (a) and (b) we arrive at a contradiction. Let us consider the case (a). Then by Definition (*denotation*) we have

(a1) $\underline{e}\ Ref\ o_1$ and (a2) $\neg\ \underline{e'} Ref\ o_1$.

From (a1) and Definition (*referring*) we obtain

(a3) $\exists u \in User \exists e \in \underline{e}\ (use(u, e) = o_1)$,

however, from (a2) and Definition (*referring*) it follows that

(a4) $\forall u \in User \forall e' \in \underline{e'} \neg\ (use(u, e') = o_1)$.

By (a3) we state that

(a5) $u_1 \in User$ and $\exists e \in \underline{e}\ (use(u_1, e_1) = o_1)$ and applying (4) to (a5), we can state that there is an expression-*token* $e'_1 \in \underline{e'}$ such that

(a6) $use(u_1, e'_1) = o_1$.

In view of (a5) $u_1 \in User$, and $e'_1 \in \underline{e'}$. Hence and from (4) we state that

(a7) $\neg\ use(u_1, e'_1) = o_1$.

Formula (a5) and formula (a7) are contradictory. The statements expressed by formulas (a5) and (6) are in a contradiction.

The proof that case (b) leads also to a contradiction, it is completely analogous. So, our theorem to be proved is justified. □

The reverse implications do not necessarily hold, because two expression-types can have the same denotation, but not every user of the language must use both of them. Hence, the first condition of Definition (*possessing the same manner of Use of types*) does not have to be satisfied, and these expressions do not have to have the same manner of *Use*. Then, according to Corollary 3, these expressions do not have to have identical meanings.

For example: the following pairs of English expressions:

- "the Morning Star" and "the Evening Star",
- "the highest mountain in Europe" and "the highest mountain in the Alps",
- "John is looking for the current Commander in Chief of the U.S. Armed Forces" and "John is looking for the current President of the United States of America",

have the same denotations, but different meanings, respectively.

To illustrate it further, we may consider the case where different terms or phrases are *Used* interchangeably to point to the same entity within a conversation or a text. For instance, both "the striker" and "Lewandowski", in the context of a football commentary, may refer to the same person, depending on how the expressions are *Used* by the speaker. This demonstrates that the object reference is determined not merely by the words themselves, but by their *Use*/usage within a particular linguistic-situational context.

The definition and properties of meaning given above concerned only a wfe-*type*. They defined its so-called *global meaning*. However, it may differ from several meanings determined by its sub-*types*, i.e. sub-classes of the set (class) of all its identifiable *tokens*. Then, the wfe-*type* has no *established meaning*.

For example: the term-*type* "key" as a set, class of all its *tokens* identifiable with the word-*token*

key

has not an established meaning because its global meaning differs from the meaning of its proper sub-type "key[1]" composed only from such *tokens* identifiable with the given above that refer only to keys for moving the bolt of a lock, and from the meaning of its proper sub-type "key[2]" composed only from *tokens* that refer to musical kyes.

The global meaning of the wfe-*type* is compatible with linguistic customs, linguistic practice and its manner of *Use* (*usage*). It is the most common dictionary meaning of this wfe-*type*, composed from meanings of its sub-t*ypes*.

As Jerzy Pelc (1971, 1977), Barbara Stanosz (1997, chapter 7) and also Joanna Odrowąż Sypniewska (2018, 2021) and Tadeusz Ciecierski (2021) rightly note, in the functional approach to natural language and in the functioning of this

language in a communication process, we are dealing with the meaning flexibility of its expressions and their variability in meaning. However, the "variability" of meanings is attributed not to a given expression-*type* but to the uses of its *tokens* in the sense of *use*. If the wfe-*type* has an established meaning in a linguistic-situational context, then all concrete *uses* of its *tokens* are the same (its *tokens* possess the same manner of *use*, the same referent), and its every sub-*type* has the same meaning. If the wfe-*type* has not an established meaning, then some uses of its *tokens* in that context are not the same, and there is at least one sub-*type* of it, whose meaning differs from its global meaning or from its definite meaning in language.

It is important to clearly distinguish the notion 'definite meaning' in language from the pragmatic notion 'established meaning' in a given linguistic-situational context. An expression-*type* with a definite meaning in a language may not have an established meaning when used by different language users in a given situation, for example, during a discussion when one of them *Uses* its sub-type, *using* tokens of that expression, in a meaning narrower than its definite linguistic meaning used by another user of the discussion. For example, when the word "congress" is *Used* in a discussion, and this word has a definite meaning in English, one participant can refer its sub-*type* only to one denotatum, the same congress, *using* certain *tokens* of this its sub-type to the same referent, the same congress, while another participant, *using* any *tokens* of this term, can refer them to any congress. The first participant then *Uses* a sub-*type* of the word “congress” in the narrow meaning, while the other one *Uses* a sub-*type* of the word “congress” in the definite meaning adopted in English, in which the word "congress" refers to any congress, and *tokens* of this word-*type* can have different referents, which are different congresses.

The formal approach to the statements given above in the theory ***MT*** is determined by the definitions and theorems given below.

DEFINITION (*possessing meaning*): $\underline{e}$ has a meaning iff

$$\exists \underline{e}' \subseteq \underline{e}\ \exists M \subseteq \underline{S}^*\ (M = \mu(\underline{e}'))\ ,$$

i.e. there exists a set of wfe-*types* which is the meaning of a sub-*type* of the wfe-*type* $\underline{e}$.

DEFINITION (*possessing an established meaning*): $\underline{e}$ has an established meaning iff

$$\forall \underline{e}' \subseteq \underline{e}\ (\mu(\underline{e}') = \mu(\underline{e})),$$

i.e.

$$\neg\exists \underline{e}' \subseteq \underline{e}\ (\underline{e}' \neq \underline{e} \wedge \mu(\underline{e}') \neq \mu(\underline{e})),$$

i.e. no proper sub-*type* of $\underline{e}$ has a meaning different from $\underline{e}$.

DEFINITION (*non-possessing an established meaning*): $\underline{e}$ has not an established meaning iff

$$\exists \underline{e}' \subset \underline{e} \, (\underline{e}' \neq \underline{e} \, \wedge \mu(\underline{e}') \neq \mu(\underline{e})),$$

i.e. there exists a proper sub-*type* of $\underline{e}$ which has a meaning different from $\underline{e}$.

From the above definitions we can conclude

COROLLARY 2:

a. *Every wfe-type of the set $\underline{S}$* has at least one meaning*,
b. *$\underline{e}$ has an established meaning iff $\underline{e}$ has one meaning*,
c. *$\underline{e}$ has not an established meaning iff $\underline{e}$ has more than one meaning*.

If users, in a conversation or discussion, *Use* an expression-*type*, e.g. the word "congress" in an established, one meaning, then they only talk about one and the same object, the same congress, using its *tokens*; these *tokens* then possess the same manner of *use*. However, if they use an expression-*type*, e.g. the *type* "congress" using its *tokens* with different references, with different congresses in mind, then this expression has not an established meaning in this conversation or discussion.[1]

The above statements follow from the following theorem of the theory ***MT***:

THEOREM:

a. *If $\underline{e}$ has an established meaning, then*

$$\forall e_1, e_2 \in \underline{e} \; (e_1, \; e_2 \; \textit{have object reference} \Rightarrow e_1 \approx e_2),$$

b. *If*

$$\exists e_1, e_2 \in \underline{e} (\neg (e_1 \approx e_2))$$

then e has not an established meaning.

[1] In the above example, the word "congress" used in the established meaning would be used as "the congress", and in the unestablished meaning as "a congress".

Proof a.

Let us assume that

(1) $\underline{e}$ has an established meaning, and additionally that

(2) $e_1, e_2 \in \underline{e}$

and

(3) e_1, e_2 have object reference.

From (2) it follows that singletons $\{e_1\}, \{e_2\}$ are sub-types of $\underline{e}$, i.e.

(4) $\{e_1\} \subseteq \underline{e}$ and $\{e_2\} \subseteq \underline{e}$.

In accordance with (1) and Definition (*possessing an established meaning*), all sub-*types* of $\underline{e}$ have the same meaning. Thus, by (4)

(5) $\mu(\{e_1\}) = \mu(\{e_2\})$.

It follows from the above Corollary3 and (5) that singletons $\{e_1\}, \{e_2\}$ have the same manner of *Use,* i.e.

(6) $\{e_1\} \cong \{e_2\}$.

From Definition (*possessing the same manner of Use of types*) and (6) we get

(7) $\forall u \in User\ \forall o \in Ont\ ((use\ (u, e_1) = o$ iff $(use\ (u, e_2) = o))$.

From the assumption (3) and Definition (*possessing object reference*), it follows in particular that

(8) there are $u_1 \in User$ and $o_1 \in Ont$ such that $use\ (u_1, e_1) = o_1$.

Applying (7) for the user u_1 and the object o_1, we get

(9) $use\ (u_1, e_1) = o_1$ iff $use\ (u_1, e_2) = o_1$

Then, from (8) and (9) it follows that $use\ (u_1, e_1) = o_1$ and $use\ (u_1, e_2) = o_1$. Thus, we can conclude that

$\exists u \in User\ (use\ (u_1, e_1)) = o_1$ and $\exists u \in User\ (use\ (u, e_2)) = o_1)$, and hence that

$\exists o \in Ont\,[\exists u \in User\,(use\,(u_1, e_1)) = o$ and $\exists u \in User\,(use\,(u, e_2)) = o]$.

So, according to Definition (*possessing the same manner of use*), wfe-*tokens* e_1, e_2 have the same manner of *use*, the same object reference, i.e. $e_1 \approx e_2$. This completes the proof of our theorem. □

Remark. The notion of non-possessing an established meaning differs from the notion of ambiguity. The latter notion is defined by means of the notion of denotation (see Wybraniec-Skardowska, 2007, 2022). Ambiguity is a sufficient condition for not having an established meaning. However, an expression which has not an established meaning does not have to be ambiguous. For example: the term "congress" may not have an established meaning in a given discussion, but it is unambiguous. 'Ambiguity' is a notion of language in general and 'possessing established meaning' is a pragmatic notion. It is also worth noting that the distinction drawn here helps clarify certain conceptual misunderstandings prevalent in semantic theory. While ambiguity and non-possession of established meaning can sometimes overlap, recognizing their difference allows conducting a more nuanced analysis of linguistic phenomena. This distinction underscores the importance of context and pragmatic factors in determining how meaning is ascribed or withheld from an expression.

5. Concluding Remarks and Comments

The concept of a functional approach to the logical theory of natural language introduced by Jerzy Pelc involves taking into account the way in which its analyzed expressions are used. Pelc takes into account two ways of using expressions and linguistic signs. In the first of them, the way of *use* takes place in specific linguistic contexts and extralinguistic situations, and it refers to sign/expression-*tokens*, material objects; and in the second – the way of *Use* (*usage*), shows the expressions of this language as sign/expression-*types*, abstract objects, detached from the context, isolated dictionary items. There are important relationships between the way of *use* in the first understanding and the way of *Use* (*usage*) in the second one. The manner of *use* in the first sense serves to determine the manner of *Use* in the second sense.

The presented general logical concept of meaning allows us to formally and precisely define the global meaning of a linguistic expression as determined by the way the expression (expression-*type*) is *Used* in the second sense; this concept refers to ideas coming from L. Wittgenstein and K. Ajdukiewicz.

The actual functioning of a linguistic expression in a specific meaning is relativized to its *use* in the first sense. The meaning-related role of an expression-*type* varies depending on its functioning in the linguistic and situational contexts. The different *tokens* of this expression *used* in such contexts mean that it can have an unestablished meaning and several meanings for users of language.

The outlined logical theory of meaning (the theory ***MT***) takes advantage of the *use*/*usage*, *Use* distinction introduced by Jerzy Pelc and shows that the basic issues of logical semiotics of natural language can be precisely defined and described using logical-formal means. At the same time, the logical conception is a contribution to deepening of the study of Pelc's semiotic creativity. Drawing inspiration from Pelc, the author develops, clarifies, and explicates his approach to the logical semiotics of natural language, generalizing and formalizing it, contributing to the construction of a general logical theory of meaning and denotation. This theory thus solves an important problem in the philosophy of language, answering the question of what the meaning of a linguistic expression is and what its denotation is.

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