

MARTINA ČÍHALOVÁ *, ZUZANA RYBAŘÍKOVÁ **

A COMPARISON OF TICHÝ'S AND PRIOR'S APPROACH TO DEFINITE DESCRIPTIONS AND THE POSSIBILITIES OF ENRICHMENT OF TICHÝ'S TREATMENT¹

SUMMARY: One of Pavel Tichý's celebrated observations was that definite descriptions do not denote individuals but individual offices. In our paper, we compare Tichý's concept of individual offices to the concept proposed by Arthur N. Prior, who differentiated between the strong and the weak 'the'. The aim of our paper is to point out parallels and differences. The differences stem in particular from their different approaches to contingency given by their accepted ontology. In addition, we would like to enrich Tichý's analysis of individual offices by including certain features of Prior's concept. The proposed outline might provide a challenge for a more detailed analysis of definite descriptions in Tichý's Transparent Intensional Logic.

KEYWORDS: definite descriptions, individual office, A. N. Prior, P. Tichý, temporal logic, Transparent Intensional Logic.

* Palacký University Olomouc. E-mail: martina.cihalova@upol.cz. ORCID: 0000-0001-9451-6523.

** Palacký University Olomouc. E-mail: zuzana.rybarikova@upol.cz. ORCID: 0000-0002-6128-1868.

¹ The topic was discussed at *XXIV. Slovak-Czech Symposium on Analytic Philosophy and The Third Context, Cognition and Communication Conference*. The authors thank the attendees of the conferences for their helpful remarks. The work on this paper was supported by the project "JG_2020_005 Times, events, and logical specification" of Palacký University Olomouc.

1. Introduction

Since Russell's famous introduction of the concept of definite descriptions and its analysis in predicate logic, the concept has become one of the prominent topics in the philosophy of language. The aim of this paper is to present concepts of definite descriptions that were postulated by Pavel Tichý and Arthur Norman Prior. Specifically, the paper proposes the enrichment of Tichý's concept with the ideas introduced by Prior. The enrichment could lead to a more precise analysis through Tichý's system of logic, Transparent Intensional Logic (further TIL).

A definite description is a phrase that refers to precisely one individual through some unambiguous mechanism. In languages that require determiners before names, this mechanism depends on the use of a definite article (Ludlow, 2018). This is the reason why Russell (1920, p. 167) famously stressed the importance of the definite article and devoted two chapters of his *Introduction to Mathematical Philosophy* to it. Russell (1920, p. 177) described the definite description on the example of the proposition 'The author of *Waverly* is Scotch'. According to Russell, the definite description is sufficient if:

- 1) there is at least one individual that wrote *Waverly*,
- 2) there is at most one individual that wrote *Waverly*,
- 3) and the individual that wrote *Waverly* was Scotch.

In the predicate logic, this could be formalized as:

$$\exists x[A(x) \wedge \forall y[A(y) \rightarrow (x = y)] \wedge S(x)]$$

Russell understood the proposition as always having the same truth value and developed a primarily extensional system of logic.

Tichý and Prior differed from Russell in the following two aspects that they have in common. It is precisely these two aspects that allow the enrichment that is presented in the second part of this paper. First, both logicians were proponents of *temporalism* (Prior, 1957, p. 8; Tichý, 1980, p. 357). Temporalism is the view that propositions could have different truth values at different times. For example, the proposition 'Today is a sunny day' could be true or false with respect to the current weather situation on a respective day. If the sun shines today and it was cloudy yesterday, it was false yesterday, and it is true today. The opposite view to temporalism is *eternalism*, whose proponents (among them Russell) argue that every proposition has a fixed truth value. It means that the claim 'Today is a sunny day' corresponds to different propositions. In the below-mentioned situation, one is from yesterday, and it is false; the second is from today and is true.

Secondly, Tichý and Prior preferred intensional systems of logic. In Tichý's case, the preference is visible even within the title of his system, which was entitled *Transparent Intensional Logic*. Moreover, TIL can operate at three levels

of context: extensional, intensional, and hyperintensional. Prior was a founder of modern temporal systems of logic that are intensional. He (1969, pp. 35–36) argued that extensional systems of logic are insufficient to grasp all the functions that logic as a tool for reasoning should have. Therefore, he proposed a system of intensional logic. There is, however, also an essential difference between Tichý's and Prior's approaches to intensionality. Tichý's TIL complies with the rules of extensional logic, and its intensionality is based on intensional objects that play an important role in the system (Jespersen, 2004, pp. 10–11).² Moreover, TIL makes it possible to explicitly distinguish and precisely define three kinds of occurrence of the meanings of expressions: hyperintensional, intensional, and extensional. These three levels of abstraction will be introduced later, e.g., in Table 1. On the contrary, Prior opted for the introduction of intensional functions into logic, but at the same time, opposed to any kind of existence of intensional objects such as possible worlds or possible entities (Prior, 1976, pp. 187–190). This difference appeared to be crucial in their concepts of definite description, as will be presented further.

To introduce the way in which Tichý's concept of individual office could be enriched by Prior's ideas, we present their concepts of definite descriptions. The order is ahistorical, as Prior's concept was postulated earlier than Tichý's. However, the system that will be enriched is introduced first. After the presentation of both concepts, the TIL is briefly introduced to be able to outline the second part, which is the innovation of this paper. In this part, we specify how Prior's ideas could be incorporated into TIL and propose three basic categories of the strong 'the'.

2. Definite Descriptions in Pavel Tichý's Logic

Pavel Tichý (1988, p. 201) distinguished three basic types of entities: classes, chronologies, and determiners. He looked upon determiners as *offices* and specifies them as

[S]omething which is occupiable by objects of a definite type. The office of the American president, for example, is occupiable by individuals, different individuals at different times. It is readily seen that an office is not fully specified by the *actual* history of its occupancy. (Tichý, 1988, pp. 201–202)

He shows this with the example of two offices, the office of the American president and the office of the inhabitant of the White House. Although these two offices have the same history of occupancy, they are certainly not identical and have different meanings. Hence, individual offices and its bearer are two distinct items and differ in their logical type. Tichý (1978a, pp. 2–3) pointed out that if

² In TIL the principles of extensionality as Leibniz's law of substitution of identities and existential generalization hold at three levels of abstraction (i.e. extensional, intensional and hyperintensional level).

the president of the Czech Republic is, unlike, e.g. Santa Clause, a real character in a certain world and time instant, it could be characterized by two features:

- 1) its character,
- 2) the individual who is a bearer of this character.

Those two features are not identical as the first is the individual office and the second at most one individual that could hold the office. Tichý (1978a, p. 250) argued that Santa Claus is not inherently fictional. Its fictionality is based on the fact that there is no individual with the characteristics of Santa Clause in the universe.

An individual is a bearer of an individual office if it fulfils the requisites of the office. Requisites are the essential properties of the individual that could hold a chosen office. For example, one of the requisites for the individual office ‘the president of the Czech Republic’ is ‘to be a citizen of the Czech Republic’. Tichý (1979, pp. 408–409) stressed that requisites describe the necessary properties of bearers of offices. The individual who is the bearer of the office ‘the president of the Czech Republic’ could be a human and a Czech citizen, but the office itself could not. There are, however, also properties that could be ascribed directly to the offices, for instance, ‘being occupied’. In various states of affairs (that is, in various possible worlds and time moments), there could be various bearers of the same individual office.

Tichý (1978b, p. 277) described different possible worlds and time instances (moments) as *intensions*. At the present moment (31st May 2022), the bearer of the description ‘the president of the Czech Republic’ is Miloš Zeman. However, ten years earlier, the bearer of the individual office was different, as could also be at the same moment but in different possible worlds. There were also time instances or possible worlds where the individual office was not occupied (for instance, 100 years ago when there was no Czech Republic but Czechoslovakia).

As was mentioned, Tichý argued that definite descriptions do not denote individuals but individual offices. One of the reasons why Tichý proposed this differentiation was his anti-contextualism. According to Tichý (1978a, p. 249), contextualism is ‘the view that the reference of lexicographically univocal terms depends on the context in which they appear’, which means that ‘[e]very denoting term, [...], is systematically ambiguous. Hence when one wants to know whether a given term stands for a given item, one must look at the term alone; for the answer may depend on the context in which the term stands.’

Tichý showed why contextualism is wrong, according to him, and how it stems from inadequate analysis of the statements. For instance, the following proposition:

- (1) The president of the Czech Republic does not like journalists

is not, according to Tichý, about the individual who occupies the office. Although the current president of the Czech Republic Miloš Zeman is known for his complicated relationship with journalists, it is not Miloš Zeman who is men-

tioned in this proposition, but the office. Tichý (1979, p. 407) argued that the subject in this type of proposition denotes the office of the president of the Czech Republic and not the concrete individual. It claims that the bearer of the office does not like journalists. However, the question of who the bearer of the office is in a chosen time and world remains open.

According to Tichý (1978a, pp. 7–9), another issue is linked with the arguments containing offices, the prevention of logical inconsistencies by proper analysis of definite descriptions as offices. The following argument:

- 1) *The president of the Czech Republic* is the husband of Ivana Zemanová,
 2) *The president of the Czech Republic is an elective office*,
 Z: The husband of Ivana Zemanová is an elective office

is not valid as the conclusion does not follow. At the time of writing this paper, both premises are true, and the conclusion is false. The invalidity of the argument could be decided with respect to the context, claiming that the subject in the first and the second premise have different denotations and, therefore, they cannot be substituted. However, this was the way that Tichý, as being anti-contextualist, aimed to avoid. He argued:

[T]he one linguistic form *the S is a P* conceals two radically different logical constructions. It would be a mistake, however, to follow contextualist's advice and put the difference down to reference shifts on the part of the subject term 'the S'. The subject term stands uniformly for the character specified by 'S'. The difference lies rather in the way the predicate *P* is applied to the subject. The predicate may be applied directly to the character spoken of (*dictus*). Such a construction may be described – stretching somewhat a medieval term – as a predication *de dicto*. Alternatively, the predicate may be applied to whichever individual thing (*res*) bears the character. This construction may be described as a predication *de re*. (Tichý, 1978a, pp. 254–255)

Tichý postulated that if the denotation is an intension, the reference is the value of this intension in the actual world at the present time. By distinguishing two ways of applying the predicate to the subject (*de dicto* and *de re* supposition), the definite description 'the president of the Czech Republic' is an unequivocal, clear, and semantically self-contained term. The term has the same denotation in both premises, the individual office. An individual office cannot be identified with its bearer, the concrete individual. Individual offices are intensions, and their output is, at most, one individual with respect to respective possible worlds and times (Tichý, 1978b, p. 277). In the first premise, the term 'the president of the Czech Republic' is used in the supposition *de re*, where the actual bearer of the office (i.e., the value of the intension in the respective possible world and time) is the subject of the predication. On the other hand, in the second premise, the object of predication is the individual office itself (i.e., the

whole intension). In the section *A Brief Introduction to TIL*, the specification of this difference in the notation of TIL is demonstrated.

3. Definite Descriptions in Arthur Prior's Logic

Prior (1967, pp. 172–174) prioritized propositional systems of logic and considered predicate calculi troublesome. It might be the reason why, unlike Tichý, he never developed a systematic theory of individuals. He was, however, very precise in listing several issues that could be linked with systems of temporal predicate logic. One issue, which he mentioned in his book *Time and Modality*, led him to the differentiation between two different meanings of the definite article ‘the’. This section introduces the issue and Prior’s solution, which could be fruitfully used as a development of Tichý’s theory of individual offices.

The previously mentioned issue concerns the Barcan formula, namely its form $\diamond\exists x\varphi(x) \rightarrow \exists x\diamond\varphi(x)$. Prior (1957, pp. 26–27, 30–34) pointed out that acceptance of the formula leads to an unwelcomed enlargement of the ontology. The issue is clearer when the Barcan formula is interpreted in temporal logic, where the operator “F” stands for “it will be the case”. Then the formula $F\exists x\varphi(x) \rightarrow \exists xF\varphi(x)$ could mean ‘If it will be the case that there exists someone who flies to Mars, then there exists someone about whom it will be the case that he or she flies to Mars’.

The formula either implies some kind of the existence of individuals that do not exist at present or is not a tautology in temporal logic. Unless any kind of existence of the currently non-existent individual is admitted and the person who will one day fly to Mars (if there will be such a person somewhere in the future) has not been born yet, the antecedent of the Barcan formula would be true, but its consequent would be false. Hence the formula would be false, too. Prior (1957, p. 32), who did not intend to include non-existent entities in his ontology, therefore excluded the Barcan formula from his system of temporal predicate logic.

However, he (1957, pp. 63–66, 77) also claimed that a system of predicate temporal logic could contain the Barcan formula if the calculus is not based on Russell’s predicate logic but on Leśniewski’s Ontology. In this Leśniewski’s calculus of names, the existential quantifier does not imply the existence of the individuals. The bounded variables a, b, c , etc. stand for the semantic category of names, which in Leśniewski’s theory means that they could be replaced by proper names as well as common nouns or even empty terms. As the particular quantifier is not linked with existence, the Barcan formula lacks its unwelcome implications in the calculus. The primitive operator (that is, the operator that appears in the axiom and due to which all other operators are introduced) of the calculus is ‘ ε ’. and the fragment of the formula ‘ $a \varepsilon b$ ’ means ‘The a is a b ’. Therefore, the first variable ‘ a ’ stands in the definition for a definite description or a proper name.

Prior (1957, p. 76) argued that in the system of temporal predicate logic based on Leśniewski’s calculus, the definite article could have two distinct interpretations. It could stand for the weak ‘the’ or for the strong ‘the’. In *Past, Pre-*

sent and Future, Prior (1967, p. 164) described the use of the weak 'the' as 'The only thing that is *now* an *a* is now a *b*' and the strong 'the' as 'An *a* which is the only thing *ever* to be an *a*, is a *b*.' The weak 'the' appears in the case that the definite description stands for individuals who could differ in time, such as 'the president of the Czech Republic', 'the Queen of England', 'the King of France', 'the winner of the Giro d'Italia', etc. In every moment, there is at most one individual to which the description refers but the reference could change in time. On the contrary, there are also definite descriptions that refer to one and the only individual in the entire history of the world as 'the first president of the Czech Republic', 'the winner of the Giro d'Italia 2022', 'the author of Marmion', etc. In this case, the definite article 'the' is the strong 'the'.

4. Comparison of Prior's and Tichý's Approach

Prior's differentiation between two meanings of the definite article and the lack of this differentiation in Tichý's TIL, otherwise precise, could be caused due to the fact that Prior unlike Tichý, did not consider different possible worlds. While the reference in Tichý's system of logic could change in time and across possible worlds, there is just the change in time in Prior's temporal predicate logic. His system is entirely in the actual world. Although Prior discussed contingency in his work, he admits it only for the propositions about the future. In the case of the analysis of present and past events, he opposed contra-factuality, as he (2003, pp. 90–92) presented, for example, in his paper *Identifiable Individuals*. Therefore, when he discussed the strong and the weak 'the', he considered just a timeline within the actual world but not various possible worlds. This might have caused that he saw more clearly the distinction between the two types of definite articles with regard to their meaning.

There is also another important difference between Tichý's and Prior's approach to definite descriptions. Namely, definite descriptions do not denote individuals, but individual offices in Tichý's TIL. In Prior's temporal predicate system of logic, definite descriptions denote individuals. We are convinced, that despite the differences mentioned above, the distinction between two different meanings of definite articles could profitably enlarge Tichý's analysis. This procedure will be presented in the following sections after a brief introduction to TIL principles.

5. A Brief Introduction to TIL

In order to include Prior's approach in Tichý's TIL, we must first briefly introduce the basic principles of TIL. For the purposes of this paper, we do not intend to include too many technical details. For those interested in TIL, we recommend, e.g., Duží, Jespersen, and Materna's (2010), where to find rigorous definitions of this system, which is still being developed by followers of Tichý.

Tichý defines the meaning of an expression as an abstract procedure (structured from the algorithmic point of view) that produces the object denoted by the expression. However, in rigorously defined cases, this procedure can fail to produce a denotation if there is none. Constructions can consist of constituents, i. e., subconstructions, which have to be executed to obtain the final output. This output can be an extensional or intensional entity or lower-order procedure.

Each object on which the constructions operate receives a type. From the logical point of view, TIL is a partial, typed lambda calculus with a ramified hierarchy of types. For the purposes of natural-language analysis, Tichý assumes the following base of ground types:

- o : the set of truth values $\{T, F\}$;
- i : the set of individuals (the universe of discourse);
- τ : the set of real numbers (doubling as times);
- ω : the set of logically possible worlds (the logical space).

These ground types can constitute functional types. And intensions are just one of them. Every empirical expression denotes α -intensions (where α is the general type of the entity, which is the value of the function after the application of the function on a possible world and time). Intensions are functions with the domain of possible worlds. In TIL, intensions are viewed as functions that map possible worlds (of type ω) to a type β . The type β is frequently the type of a chronology of the elements of type α . These α -chronologies are, in turn, functions mapping time (of the type τ) to the type α . Thus, α -intensions are usually mappings of type ω to type τ , and to type α , or in the TIL notation $((\alpha\tau)\omega)$, $\alpha_{\tau\omega}$ for short.

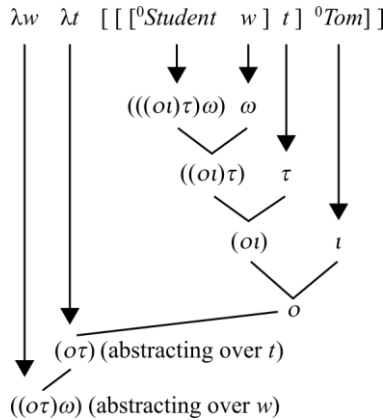
Let's have, for instance, the intensional objects as properties of individuals. Properties are objects of type $((o\tau)\omega)$, i. e. of type $(o\tau)\omega$ for short. In order to apply a property to an individual, a functional application is used. However, properties are not type-theoretically proper entities to be directly applied to an individual. They have to be extensionalised first. For instance, the sentence 'Tom is a student.' ascribes the property of being a student to Tom. As any other non-procedural objects to be operated on, the individual Tom, as well as the property of being a student, are supplied by their Trivialization, ${}^0\text{Tom}$, ${}^0\text{Student}$. The sign 0 here indicates a construction named *Trivialisation*. A Trivialisation presents an object X without the mediation of any other procedures.³ Trivialisation can present an object of any type, even another construction C .⁴ Since the type of property is

³ Using the terminology of programming languages, the Trivialisation of X , denoted by 0X , is just a pointer or reference to X .

⁴ Tichý defined six kinds of meaning procedures and called them constructions. There are two kinds of atomic constructions that present input objects to be operated on by molecular constructions. They are *Trivialisation* and *Variable*. There are two kinds of molecular constructions, which correspond to λ -abstraction and application in the λ -

an intension of type $((oi)\tau)\omega$, or $(oi)_{\tau\omega}$ for short, the property must be applied to a possible world (type ω) first and then to time (type τ), i. e. extensionalized. We call this extensional descent to the respective possible world and time. To this end, we have variables w with the domain of objects of type ω and t with the domain of objects of type τ . Thus, we get $[[^0Student\ w]\ t]$, or ${}^0Student_{wt}$ for short. In this way, we obtain the population of students in the world w and time t in which we are going to evaluate the truth-value of the sentence. That Tom belongs to this population is expressed simply by the application of this population to Tom: $[{}^0Student_{wt}\ ^0Tom] \rightarrow o$.⁵ Finally, we abstract over the values of the variables w and t to obtain the proposition that Tom is a student: $\lambda w \lambda t [{}^0Student_{wt}\ ^0Tom] \rightarrow o_{\tau\omega}$. For a better understanding, there is a type-theoretical checking below, where we verify that the construction has been combined in a type-theoretically coherent way.

Figure 1



It will now be shown how TIL can operate at three levels of abstraction to specify above mentioned *de dicto* and *de re* suppositions. Let's have the expression 'the King of France is bald' which expresses a procedure, i.e. construction. When one uses this construction to produce its output as its denotation, one obtains a proposition, i.e., intension whose truth value depends on the respective possible world and time. However, one can investigate the actual truth value of

calculi, namely *Closure* and *Composition*. And finally, the special TIL construction *Double Execution* is needed because TIL is a hyperintensional system and each construction can occur not only in execution mode so as to produce an object (if any), but also as an object in its own right on which other (higher-order) constructions operate. Those interested in the exact specifications of modes of forming constructions are referred to Tichý (1988, pp. 63–76).

⁵ The sign “ \rightarrow ” expresses that the construction displayed on the left constructs the object of the type displayed on the right. Read this example as: the construction $[{}^0Student_{wt}\ ^0Tom]$ constructs the object of type o (a truth value).

this proposition in the particular world and time. Hence, in the intensional level of denotation, this construction produces a proposition which is an intension, and in the extensional level of denotation, it produces a truth value. In the table below, there are specified three levels of abstraction and a difference between two types of supposition *de dicto* (intensional use of construction) and *de re* (extensional use of construction). This table is based on the figure in Duží, Jespersen, and Materna (2010, p. 233).

Table 1*Three levels of abstraction*

Expression <i>E</i> can be			
At the linguistic level	mentioned	used to express its meaning	
At the conceptual level		procedure (TIL construction <i>C</i> of <i>E</i>)	
		<i>C</i> can be mentioned on <i>hyperintensional</i> level	<i>C</i> can be used to produce its output on <i>intensional</i> or <i>extensional</i> level
At the denotational (functional) level		<i>CE</i> intensional using (<i>De dicto</i> supposition)	<i>CE</i> extensional using (<i>De re</i> supposition)

The above-mentioned construction of the expression ‘the King of France is bald’ contains sub-constructions of the property of ‘being bald’ and of an individual office ‘the King of France’. The construction of property requires an individual to operate. However, the individual office ‘the King of France’ is not occupied in this actual world and time and fails to produce the output at the extensional level, which may be a unique individual. Hence, the whole construction fails to produce the output on an extensional level. In TIL jargon, it is *improper*.

The expression ‘the King of France is bald’ denotes a proposition of type $\sigma_{\tau\omega}$, and comes down to this TIL construction:

$$\lambda w \lambda t [{}^0 \text{Bald}_{wt} {}^0 \text{King of France}_{wt}]$$

The lower indexes ‘*wt*’ suggest intensional descent, that is, the execution of sub-construction on the extensional level to produce its output. The type of denoted entity of the property ‘being bald’ is $(\sigma)_{\tau\omega}$. This is the function with arguments from the domain of possible worlds, times, and individuals. The output of this function is a truth value according to which an investigated individual is bald or not. The type of denotation of the individual office ‘King of France’ is $\iota_{\tau\omega}$.

This is the function with arguments from the domain of possible worlds and times, which produces a unique individual, if any.

In the actual possible world and time, the construction $[{}^0\text{Bald}_{wt} {}^0\text{King of France}_{wt}]$ denotes no truth value, i.e. there is a value gap. To construct the proposition, one has to abstract over variables of possible worlds and times, as is usual in lambda calculi, to obtain the above construction $\lambda w \lambda t [{}^0\text{Bald}_{wt} {}^0\text{King of France}_{wt}]$.

Let us now return to the argument discussed in Section 2 to analyze it in TIL:

- 1) *The president of the Czech Republic* is the husband of Ivana Zemanová,
 2) *The president of the Czech Republic* is an elective office,
 Z: The husband of Ivana Zemanová is an elective office

contains the types of entities as follows:

Ivana Zemanová / i , individual,
Husband_of / $(ou)_{\tau\omega}$, the relation in intension between two individuals,
The president of the Czech Republic (PCR in brief) / $i_{\tau\omega}$, individual office,
Elective / $(\alpha \ i_{\tau\omega})_{\tau\omega}$, the property of individual offices.

The analysis of premises in TIL is as follows:

1. $\lambda w \lambda t [{}^0\text{Husband}_{wt} [{}^0\text{PCR}_{wt} {}^0\text{IvanaZemanová}]]$
2. $\lambda w \lambda t [{}^0\text{ElectiveOffice}_{wt} {}^0\text{PCR}]$

With this specification, it is evident that the substitution of the term ‘the husband of Ivana Zemanová’ instead of the term ‘president of the Czech Republic’ in the second premise is not correct. The construction of the president of the Czech Republic in premise one is used extensionally as opposed to the construction in premise two, where it is used intensionally. Therefore, the conclusion does not follow and this argument is invalid.

After this brief introduction to the basic principle of TIL, we will introduce in the following paragraphs the enrichment of Prior’s distinction of strong and weak ‘the’ within the framework of Tichý’s system of logic.

6. The Inclusion of Prior’s Ideas in Tichý’s TIL

According to Prior, in the case of the weak ‘the’, the denotation of the definite description is at most one, but it could change over time. Example of a sentence with a weak ‘the’ is, for instance, ‘The president of the Czech Republic does not like journalists’. According to Tichý, the expression ‘The president of the Czech Republic’ denotes an individual office, the entity of type $i_{\tau\omega}$.

In the case of the strong ‘the’, the denotation of the definite description is at most one and does not change over time, as in the example of the sentence ‘The first president of the Czech Republic was a dramatist’. The type of entity denoted by the expression ‘The first president of the Czech Republic’ is ι_ω , because the value of this function (respective individual) does not depend on time.

We made a list of various examples and tried to find more general features to form a primary classification of these terms. We can distinguish the three following basic categories of the strong ‘the’:

1. The unique sequence between bearers of some individual office:
first (second, third, ...) president of the Czech Republic/pope/king of France
2. The unique sequence between bearers of some property:
the first (second, ...) child born in the respective year
the first follower of Tichy’s ideas
3. The uniqueness of the product of an individual (founder, inventor, author, murderer):
the founder of Transparent Intensional Logic
the inventor of contact lenses
the author of Waverly
the murderer of John Lennon

In the previous example of the term ‘The first president of the Czech Republic’, the construction [${}^0\textit{First}$ ${}^0\textit{PCR}$] has to construct the entity of type ι_ω (in a given possible world, it will be at most one individual). The construction ${}^0\textit{PCR}$ constructs the individual office, the entity of type $\iota_{\tau\omega}$. However, what type of object constructs the construction ${}^0\textit{First}$?

The term ‘first’ modifies the individual office of the president in the sense that the ‘first Czech president’ is the president and is, moreover, the first of the Czech presidents. Duží, Jespersen, and Materna (2010, p. 395–406) distinguish important concepts called modifiers of intensions. The intension modifier most often modifies a property; however, it can also modify an individual office. In the sentence ‘If a is a skilful surgeon, then a is a surgeon’, the term ‘skilful’ plays the role of a *modifier* of the property of being a surgeon. Property modifiers are functions that assign to a core property another modified property; as in the case of the concepts of ‘skilful surgeon’ and ‘small elephant’ where ‘skilful’ and ‘small’ are modifiers of the properties of being a surgeon and of being an elephant. The type of modifier is thus as $((\textit{OI})_{\tau\omega}(\textit{OI})_{\tau\omega})$, i. e. it is the function that applied to some property (intension) and gives a slightly modified property (intension) as its output.

Duží, Jespersen, and Materna distinguish several categories of modifiers based on how they modify the property (or intension in general) to which they are applied and what could or could not be derived. In the case of *subsective* and

intersective modifiers, the following rule is valid: if a is an AB , then a is B . Hence, a skilled surgeon is a surgeon and a small elephant is an elephant. However, in the case of so-called *privative* modifiers (as 'wooden' and 'fake' in the expressions 'wooden horse' and 'fake banknote'), this rule is not valid (wooden horse is not a horse, and a fake banknote is not a banknote).

Below, there are introduced the differences between subsective and the intersective category of modifier concerning a further derivation in connection with another property. The major difference between subsective and intersective modification is that subsectivity bans this sort of argument: Premise 1: $AB(a)$, Premise 2: $C(a)$, Conclusion: $AC(a)$. (Where B and C are distinct properties, A is an intensional modifier, a is individual). Duží, Jespersen, and Materna make the following distinction in the context of the above examples of 'skilful surgeon' and 'small elephant':

Charles may be a skilful surgeon, and he may be a drummer too, but this does not make him a skilful drummer. Scalar properties are subsective modifiers. Again, Jumbo may be a small elephant, as well as the mammal, but this does not make Jumbo a small mammal. (Duží, Jespersen, Materna, 2010, p. 399)

Let us now return to the discussed type of object that is constructed by the construction ${}^0\textit{First}$ in the previous example of the term 'The first president of the Czech Republic'. The type of *First* is a subsective modifier of office because it holds that:

- if the first Czech president is an individual a , then a is the Czech president,
- if an individual a is the first Czech president, then a is the first (from Czech presidents).

The type of *First* is $(\iota_{\omega}\iota_{\tau\omega})$. It is the function applied to some individual office (intension) that gives as its output a slightly modified office whose occupation does not depend on a time variable (the category of strong 'the' according to Prior).

Due to its high expressive power, TIL allows the specification of terms that denote the strong 'the'. However, this rigorous specification requires a fine-grained analysis of all entities included as sub-construction of these constructions of such terms and the definition of the concept identifying the strong 'the'. Thus, it is necessary, for example, to define in TIL constructions the concept of 'The first president of the Czech Republic' as the president who was the first of all Czech presidents in time. This is far from trivial since it is necessary to define a function that selects the interval which is the first one from all the intervals in which the Czech president existed, and then it selects the individual which is the president in such an interval. Our future work is thus to specify a precise definition of concepts such as 'the first Czech president', 'the murder of Abraham Lincoln', etc., which represent the strong 'the'. The first step toward this precise specification was the classification of the general types of strong 'the', which is

one of the novelties of this paper. Each category of this classification then requires a general pattern for all the particular instances of the strong ‘the’ that could be derived.

6. Conclusion

In our paper, we compared Tichý and Prior’s concepts of definite descriptions. Despite differences, these two concepts shared several similarities that, as we demonstrate, could allow the enrichment of the analysis in Tichý’s TIL by ideas from Prior’s temporal predicate system of logic. Our paper is, however, the first outline, which needs to be further elaborated at the level of concrete specification of general patterns of the strong ‘the’ in TIL. There is also the question of whether the three categories we propose to distinguish cover all examples of the strong ‘the’. The issue was introduced and we would definitely like to develop it in our future work.

REFERENCES

- Duží, M., Jespersen, B., Materna, P. (2010). *Procedural Semantics for Hyperintensional Logic. Foundations and Applications of Transparent Intensional Logic* (1st ed.). Berlin: Springer.
- Jespersen, B. (2004). The Foundations of Tichý’s Logic. In V. Svoboda, B. Jespersen, C. Cheyne (Eds.), *Pavel Tichý’s Collected Papers in Logic and Philosophy* (pp. 9–23). Prague: Filosofia; Dunedin: University of Otago Press.
- Ludlow, P. (2018). Descriptions. In E. N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy*. Retrieved from: <https://plato.stanford.edu/archives/fall2018/entries/descriptions/>
- Prior, A. N. (1957). *Time and Modality*. Oxford: Clarendon Press.
- Prior, A. N. (1967). *Past, Present and Future*. Oxford: Clarendon Press.
- Prior, A. N. (1976). Intentionality and Intensionality. In A. N. Prior, *Papers in Logic and Ethics* (pp. 187–201). London: Duckworth.
- Prior, A. N. (2003). Identifiable Individuals. In P. Hasle, P. Øhrstrøm, T. Braüner and J. Copeland (Eds.), *Papers on Time and Tense* (pp. 81–92). Oxford: OUP.
- Russell, B. (1920). *Introduction to Mathematical Philosophy* (2nd ed.). London: George Allen and Unwin.
- Tichý, P. (1978a). De Dicto and de Re. *Philosophia*, 8, 1–16.
doi:10.1007/BF02380175
- Tichý, P. (1978b). Questions, Answers, and Logic. *American Philosophical Quarterly*, 15(4), 275–284.
- Tichý, P. (1979). Existence and God. *The Journal of Philosophy*, 76(8), 403–420.
doi:10.2307/2025409
- Tichý, P. (1980). The Logic of Temporal Discourse. *Linguistics and Philosophy*, 3(3), 343–369. doi:10.1007/BF00401690

- Tichý, P. (1988). *The Foundations of Frege's Logic*. Berlin, New York: Walter de Gruyter.
- Tichý, P. (2004). *Collected Papers in Logic and Philosophy*. In V. Svoboda, B. Jespersen and C. Cheyne (Eds.). Prague: Filosofia; Dunedin: University of Otago Press.

