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How contingent and how a priori are contingent a priori truths?

Abstract In the presented article, I have analyzed the famous Saul Kripke statement that some a priori truths are contingent. I show, that despite Kripke's thesis, in the historical understanding of contingency, the notions of contingency and apriority are in deep conflict with each other. In this understanding of contingency, the past, which can be known a priori, is not contingent, and the future, which is contingent, has difficulty acquiring a priori knowledge. Having stated Kripke's thesis more precisely, I propose three means in order to defend it in the historical understanding of possibility: (a) by introducing the notion of "factual" future, (b) by replacing the notion of apriority with the notion of apriority-in-the-future; (c) by replacing the notion of apriority with the notion of historical apriority, and the notion of contingency with the notion of once-apriority. In the annex of the article, I present the formal analysis of the problem that I have introduced and three solutions which I have proposed in the language of temporal-modal logic of predicates for models of indeterministic time.

Keywords a priori, contingency, modality, time

1. Introduction

In his famous work, *Naming and Necessity* (1980), Saul Kripke provided several examples meant to refute the traditionally sanctioned thesis that all a priori propositions are at the same time necessary.[2] The best-known is

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[2] References to relevant classic works can be found in (Turri 2011).

related to the notion of one meter (Kripke 1980) and can be paraphrased in the following way:

> Pursuant to the decision of the 1875 Meter Convention, a stick was placed at the International Bureau of Weights and Measures in Sèvres near Paris: let us call this stick S. The length of the stick S was to serve as the standard unit for the measurement of length. Moreover, at the time S reached its place, say at noon, November 22, 1875 (let us call this moment *t*), the following words were uttered: "May 'meter' refer to the actual length of the stick S at this time." Let us further assume, for the sake of the example, that the subject *L* knows this convention and uses the word 'meter' to indicate precisely this length.[3]

Let us consider, in the context of this example, the sentence uttered by L in 2016:

(A) At time *t* the stick S is one meter long.[4]

Kripke would like to persuade us that *L* should be convinced about the truth of this sentence a priori. After all, *L* knows that the length of the stick S at *t* was used to determine the reference of the word 'meter' and thus can accept the truth of (A), as Kripke puts it, "automatically, without further investigation" (Kripke 1980). The subject *L* needs nothing more than knowledge regarding the use of language in order to accept the truth of this statement. This seems to be a sufficient reason to assume that *L* can know a priori the truth of the proposition expressed by (A). At the same time, Kripke argues that the sentence expresses a contingent proposition. After all, the sheer fact that the stick S was used in certain ceremony does not make it immune to the workings of the laws of nature – had the stick been previously exposed to heat or stretched a bit, it would have been slightly longer at time t than one meter.

Kripke's example sparked off a lively debate in the philosophy of language, metaphysics, and epistemology. In this text, I would like to join the

[3] In reality, since 1960 the distance of one meter has been tied to the specific length of the light wave, not the Paris prototype.

[4] I use this tenseless version of the sentence in order to avoid additional complications stemming from tense analysis. Moreover, the example is usually presented in the form of an implication: "If the stick S exists at time *t*, then it is one meter long at *t*." To make reception easier, I will keep the non-conditional formulation of the sentence (A), especially since, according to the formal analysis I present in the Appendix, this sentence need not be subject to the condition.

debate with an observation. Given a certain understanding of the notions of possibility and contingency, Kripke's examples miss the point! Namely, it turns out that under this understanding, a priori knowability of the truth of these examples requires, as an essential condition, that *they should not be contingent*. It would thus seem that, against Kripke's claim, the notions of contingency and apriority are in conflict.

The notion of possibility that seems to exclude apriority is temporal possibility or, as I will call it, historical possibility. I mean here the notion we have in mind when we say that in 1919 it was still *possible* to avoid the Second World War, although in 1938 there was no such *possibility*. Should we want to apply a notion dual to possibility in this sense of the term, we would say that something is inevitable, irreversible, or settled. This notion of possibility is essentially connected with time – the flow of time narrows down the range of possible outcomes. In particular, the past is beyond the domain of open possibility: all that has passed is settled and irreversible. In presenting my argument, I will stick to this intuitive understanding of historical possibility. Its formal explication can be found in the Appendix (Section 6.1.). I believe that the use of this notion of possibility is appropriate in the context of Kripke's analysis since the notion of time and its interaction with possibility plays an extremely important role in his examples. In order to properly understand the meter example, for instance, it is essential to take into account when the reference of the word 'meter' is fixed, when this word is used, and which possibilities remain open and at which times.

Finally, I would like to note that my goal is not to point to some fundamental error in Kripke's argumentation. On the contrary, I intend to propose ways in which Kripke can defend himself against my objection. My aim is to elucidate the thesis about the existence of contingent a priori truths by analyzing it in light of the notion of historical possibility. As it turns out, apriority is strictly connected with historical necessity, which sheds new light on Kripke's thesis. I think that such exegesis allows for a better understanding of Kripke's text and the notions of contingency and apriority employed in the contemporary philosophy of language.

The article has the following structure: In Section 2, I cite several ways in which Kripke's thesis has been criticized to date. In Section 3, I present my own objection regarding the validity of his thesis. In Section 4, I propose two lines of defence of Kripke's thesis against my objection: by introducing the notion of actual future (4.1.) and by modifying the definition of certain key concepts (4.2.). Section 5 is a summary of the discussion. In the last part of the text I paraphrase the discussion in the formal setting of branching time.

2. Several Problems with Kripke's Example

I will begin by presenting several attempts to undermine Kripke's thesis about the existence of contingent a priori truths (the list is likely not exhaustive). This reconstruction is motivated by historical interest only, since my own argument differs from all those presented below (it is closest in spirit to the theses by Soames and Donnellan). A reader not interested in the historical attempts to undermine Kripke's thesis may skip this part without any loss to the consistency of the reading.

Let us begin with the criticism of Kripke's example presented by Chakravarti (1979). Chakravarti notices, in my opinion rightly, that in order to do justice to Kripke's intentions, the sentence (A) should be understood in the following way:

(A)$^{\text{Chak}}$ The length of the stick S at time t is the same as the length of the stick S at time t in our world.

Subsequently, the author argues that it is not possible to establish a priori whether the statement expresses a true proposition since it is not possible to establish a priori whether the description "the length of S at t" which opens the sentence (A$^{\text{Chak}}$) refers to the length of S in our world. According to Chakravarti, a context of utterance (in our world) is thinkable in which this description would refer to the length of the stick S in some other possible world. The example he brings up is of two people watching, in our world, a television program broadcasting a ceremony whereby the stick S (here characterized by slightly different parameters) is placed at a relevant location in another possible world. Chakravarti argues that in this situation the description refers to the length of the stick S in the 'transmitted' world and not in our world. As a result, the proposition expressed by the sentence (A$^{\text{Chak}}$) is false and so cannot be the subject of knowledge, including a priori knowledge. If we allow this kind of context, then in order to render Kripke's example proposition knowable a priori, we would have to explicitly add to the first description the following reference to our world:

(A)$^{\text{Chak2}}$ The length of the stick S at time t in our world is the same as the length of the stick S at t in our world.

The proposition expressed by the above sentence is undoubtedly knowable a priori, but it is no longer contingent. Therefore, once we question the assumption that the tacit reference of signs used in our world is in our world, Kripke's thesis becomes unjustified. It may be worth noting here that Kripke would probably disagree with this criticism since the above counterargument presupposes an ontology and epistemology of possible worlds radically different from the one accepted by him. Kripke writes explicitly that "a possible world isn't a distant country that we are coming across, or viewing through a telescope" (Kripke 1980: 44).

Another line of criticism has been proposed by Casullo (1977). He is of the opinion that the notion of reference-fixing description Kripke wants to use for the sake of his example is incomprehensible and vague. Following Donnellan (1966), he distinguishes two understandings of the notion of definite description: attributive and referential. According to Casullo, Kripke's examples do not express contingent a priori propositions in either of these senses. If the description "the length of the stick S at time *t*" is used attributively, then the term 'meter' is equivalent to it, and the sentence "The stick S at time *t* is one meter long" is an analytic necessary truth. If the description is used referentially, on the other hand, the reference of the word 'meter' is a particular length meant by the speaker while uttering the description. In the latter scenario, however, there is no guarantee that the length meant by the speaker is the same as the length of the stick S at *t* since this can at best be established a posteriori. One way or another, the sentence (A) does not express a contingent a priori proposition.

BonJour (1998) and Turri (2011) criticize Kripke in a mutually similar and concise manner. Namely, they hold that Kripke conflated two theses: the necessary a priori thesis that any object used to fix a unit of length will measure precisely one such unit at the time of the fixing, and the thesis that a particular object, here the stick S, at time *t* is one meter long – this thesis is contingent and a posteriori since it is impossible to establish a priori that S and not another object was used at time *t* to determine the length of the 'meter' unit.

An interesting line of criticism has been offered by Scott Soames (2003). According to Soames, Kripke's examples are misleading because their author falsely suggests that we can always use a reference-fixing description (for example, "the length of the stick S at time *t*") to express a singular proposition[5] and be convinced as to what this proposition says (that we can possess a belief about an object O that it has a property P). Soames is of the opinion that, in order to understand and believe such an individual proposition, it is

[5] Although Kripke himself did not use this terminology, Soames (2003: 400–401) argues that this thesis can reasonably be attributed to him.

necessary to be acquainted with the reference of the appropriate description and to possess an independent rationale in favour of the thesis that the description is true of the object O. According to him, the rejection of the latter assumption leads to absurd consequences. One of them is revealed in the observation Soames attributes to Kripke himself (Soames 2003: 411; Kripke is supposed to have cited it during his Princeton seminars convened after the lectures recorded in *Naming and Necessity*). If we assume that the subject L shares a false conviction expressed by the sentence S, then L is inclined to accept the proposition expressed by the sentence "The sole object x such that (if S, then x is Princeton University, and if not-S, then x is Kripke's left toe) is a university." As a result, if we are allowed to use the above sentence to express a singular proposition toward which we might hold this or that attitude, we come to the following conclusion: given that L does believe that S, L is convinced that the object which is Kripke's left thumbnail is a university. According to Soames, Kripke himself considered this example to be the *reductio ad absurdum* of the thesis that a definite description can always be used to fix the reference. However, if using a definite description to fix the reference requires acquaintance with the appropriate object and its properties, we cannot know the proposition given by Kripke a priori. Hence, Kripke's example does not illustrate a contingent truth knowable a priori.

A similar conclusion had been drawn earlier by Keith Donnellan (1977). In contrast to Soames, Donnellan believes that rigidified descriptions can be used to express singular propositions. However, he claims, this time in agreement with Soames, that one cannot hold attitudes toward propositions expressed using this instrument. In particular, one cannot know, hold or believe such propositions. Hence, Kripke's meter example does express a contingent proposition, but this proposition cannot be known without the knowledge of the actual reference of the appropriate description. In particular, it cannot be known a priori.

Many examples of contingent a priori truths have emerged in the literature since the publication of Naming and Necessity. Every example known to me, however, uses one of the following two methods of constructing truths of this type: (a) φ if and only if actually φ; and (b) the sole object possessing the property P is the sole object actually possessing the property P. The objections I raise below apply *mutatis mutandis* to all examples constructed using any of these two methods.

3. Apriority and Historical Contingency: When the Stick S Can Measure One Meter, and When It Cannot

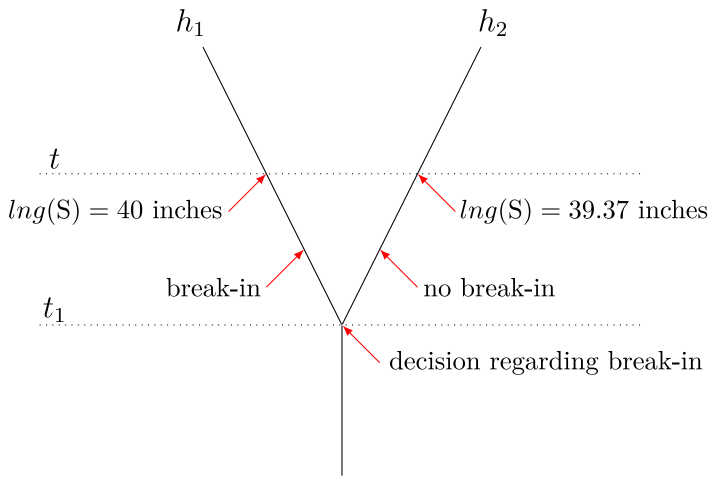
As I have indicated above, the notion of possibility I will use is historical possibility. Further below I will introduce a formal semantics of this notion; meanwhile, the intuitive understanding sketched in the Introduction should suffice. In order to move from the notion of possibility to that of contingency, I will adopt the traditional definition according to which a proposition φ is contingent insofar as it is possible that φ and it is possible that not-φ.

As I have already mentioned, given the understanding of possibility I will refer to, the future is the only domain of open possibilities, while the past is fixed and irreversible. Therefore, the future is the only domain of contingency as well. This view is by no means novel. It was voiced by Aristotle; it was also implicitly held by many theologians who discussed the (perhaps illusory) conflict between human freedom and divine omniscience (see Øhrstrøm, Hasle 1995). Another philosopher who claimed that there is a modal asymmetry between the past and the future was Charles Sanders Peirce (1958: 5.459). In Poland, similar views were articulated at some point by Tadeusz Kotarbiński (1913) and Jan Łukasiewicz (1961). The philosopher who formalized this intuition in the language of modal logic and used relational semantics for its explication was Arthur Prior (1967). Interestingly, in his formal considerations regarding historical modality Prior was inspired by Kripke himself (Ploug, Øhrstrøm 2012; Øhrstrøm, Hasle 1995: 189). As a teenager attending high school in Omaha, Nebraska, Kripke, stimulated by the book Time and Modality, wrote a letter to Prior in which he proposed a novel way of modelling the notion of historical possibility. This notion, after proper adaptation, turned out extremely fruitful for Prior and those continuing his work. The idea that possibility has a temporal dimension was thus not alien to Kripke.

Now that we are equipped with the notion of historical contingency, let us look more closely at Kripke's example. Let us focus on the sentence (A) "At time *t* the stick S is one meter long" uttered today. Anyone familiar with the appropriate convention can know the truth of the proposition expressed by the sentence a priori.[6] Let us now consider whether the sentence expresses a contingent truth. The proposition says that in 1875 a certain stick possessed a certain property. As long as we accept the notion of contingency proposed

[6] I set aside the concerns recounted in the previous section.

above, the sentence (A) is not at all contingent today. On the contrary, how long the stick S was nearly 140 years ago is an irreversible fact. It is not a contingent truth since nothing can be done about it anymore. Today we can heat up the stick S and stretch it all we want (as long as we are allowed access by the management and security at Sèvres), yet we will not be able to change its length on November 22, 1875.



Graph 1. Possible fate of the stick S.

It is thus clear that today the sentence (A) does not express a (historically) contingent proposition. Here, someone might rightly protest that this is not at all what is meant. Of course, the past length of the stick S cannot be changed today, but this has not always been the case. For example, at noon, November 21, 1875 (let us call this time t_1) the length of the stick S on the following day was still an open matter.[7] It may have so happened then that a dishonest competitor of the firm Johnson Matthey, which actually cast the meter prototype, should have commissioned a break-in into the International Bureau of Weights and Measures and a slight stretching of the stick S to the length of 40 inches. This way, the competitor would have undermined the professionalism of the British firm (the stick at Sévres would have been longer than other specimens provided by Johnson Matthey and its length different from what had been agreed, etc.). The break-in never took place but until t_1 this possibility was still open. Graph 1 illustrates the situation described by me just now. The more mundane reasons for a

[7] This is clearly the case on the condition that our world is not fully deterministic.

change in the stick's length could have been indeterministic fluctuations in temperature near Paris and their impact on the density of metal. Let us concede then that on November 21, 1875, that is at t_1, it was still contingent whether on the following day the stick S would be one meter long.

Let us now consider whether at time t_1 any person might have known a priori that on the following day the stick S would be one meter long. Such a hypothesis would be completely breakneck. In order to know that on the following day the stick S would be one meter long (i.e., circa 39.37 inches) that person would have had to be able to exclude the possibility of any break-in, to predict the precise temperature at noon the day after, etc. It is highly unlikely that anyone could be in possession of this kind of knowledge. It would be even more unusual to expect anyone to possess this kind of knowledge a priori. Much more is needed to establish such facts than basic information regarding the use of the English language. One might even argue that if the length of the stick S depended on indeterministic factors, then its prediction was not possible at all. It thus turns out that on November 21, 1875, when the length of the stick the day after was still contingent, it was not possible to know it a priori. This observation allows us to note that the notion of apriority employed by Kripke has an important temporal component: today the truth of (A) is knowable a priori, but it was not so before t_1 when the length of the stick S was still an open matter.

In light of the above argumentation we may say that by the time we know a priori that at noon, November 22, 1875 the stick S was one meter long this is no longer a contingent truth. And conversely, as long as the length of S at noon, November 22, 1875 was contingent, no one could predict a priori that it would be one meter. This observation shows that the necessary condition for Kripke style propositions to be knowable a priori is that they are historically necessary. It clarifies the dependencies obtaining between the epistemic notion of apriority and the metaphysical notion of possibility.

4. How to Defend Kripke?

I would like to propose two natural ways of overcoming the problem indicated above. The first one shows that, contrary to the appearances, the truth of the proposition expressed by (A) is knowable a priori even when it is historically contingent. In the second one, I admit that a priori truths are not historically contingent but I claim that certain modifications to the notion of either contingency or apriority allow one to express theses similar to those held by Kripke.

4.1. Actual Future and Historically Contingent A Priori Propositions

The first idea is to argue that the truth of the proposition expressed by the sentence (A) "At time t the stick S is one meter long" is knowable a priori at time t_1, when this length is still undetermined and thus contingent.

Let us go back to November 21, 1875 and assume that it is already decided that on the following day the reference of the word 'meter' will be fixed as the length of the stick S. So far I have argued that it is not possible to know a priori precisely how long the stick S will be due to the impact of indeterministic factors. However, one might like to address the objection like this: I certainly cannot predict the exact future length, for example in inches, of the stick S, but it will definitely have some particular length and thus I can fix, already today, the reference of the word 'meter' by reference to the length the stick S will actually have at time t. It is unfailingly true that the length of the stick S at time t will be equal to the actual length of S at t and so I can establish, without recourse to experience, that the sentence (A) expresses a true proposition and thus that its truth is knowable a priori. By arguing in the above manner, we accept the following (rough) definition of apriority:

Definition 1. It is knowable a priori that a sentence φ expresses a true proposition if and only if, regardless of any circumstances, when the sentence φ is uttered, it expresses a true proposition.

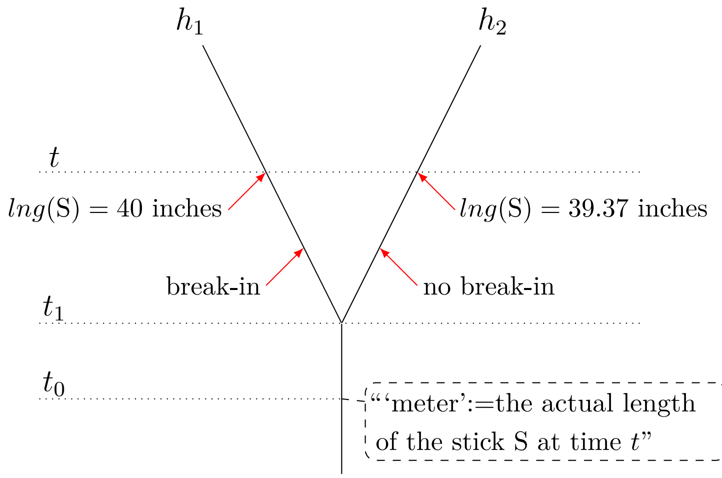
This definition is precise enough for my purposes, although it is likely not fully adequate. The implication from left to right can be questioned by reference to sentences expressing innate truths, intuitively known truths, insight or revelation – it could be argued that their truth is knowable a priori despite the fact that in some circumstances they could express false propositions (see e.g. Kripke 1980: 40, footnote 11; Turri 2011). The implication from right to left, on the other hand, could potentially be undermined (as has been done by Davies and Humberstone 1980) by reference to sentences expressing necessary propositions the truth of which is known a posteriori, such as "George Orwell is Eric Arthur Blair." I do not take it upon myself to defend the correctness of the above formulation here and if anyone has doubts regarding the proposed definition, I will say that it explicates a certain technical sense of apriority. I will keep this formulation since it is simple and sufficiently precise to meet the needs of this article; it also allows for a simplification of the formal argument I present in the Appendix.

The notion of apriority I adopt here is very close to that proposed by Robert Stalnaker (1978: 83). A similar notion has also been suggested by David Kaplan (1989b: 538, 550, 597). The implication from left to right in the above definition is accepted, in a particular formulation, by Davies and Humberstone (1980). Using the conceptual apparatus proper to two-dimensional semantics of David Chalmers (2005) we may say that the truth of the proposition expressed by a sentence φ can be known a priori if and only if 1-intension of φ yields the truth in all situations in which it is defined (Chalmers himself has doubts whether this characteristic can be identified with apriority).

Definition 1 clearly indicates that what is knowable in Kripke's examples is *the fact that the proposition expressed by the given sentence is true* and not the proposition thus expressed itself. I will not try to undermine this notion of apriority. Instead, I would like to raise the question of whether we can, accepting the above definition, establish the truth of the proposition expressed by (A) a priori. The answer to this question depends on the semantic value ascribed to the rigidified definite description constituting the definiens of the expression 'meter', that is to say, the description "the actual length of the stick S at time t." In particular, it depends on whether we assume that, given that the future length of the stick S is not determined, the description refers.

Let us take a look at Graph 2 – the diagram clearly shows that at the time when the reference is fixed (that is at time t_0) there are many possibilities as to the length of the stick S at time t. It could be 39.37 inches, but 40 inches is also possible, if the break-in to the Sèvres Bureau does take place. However, 'meter' is supposed to refer to the length the stick S will actually have. For this to make sense, one has to assume that, among the many possible future scenarios, the one which will actually occur can be distinguished. Only on this assumption can the description used at time t_0 refer to one strictly determined quantity.

Many outstanding theoreticians exploring the relationship between determinism and time question this possibility (see, among others, Belnap et al. 2001; MacFarlane 2008; Placek, Belnap 2012). They think that to speak of 'actual' or 'real' future is not valid in light of indeterminism. There is no way to tie the word 'actually' to any one possible future and we are forced to admit that it refers to an entire bundle of possible continuations allowed by the current state of the world. Hence, in the approach proposed by Belnap et al. (2001) and MacFarlane (2008) the actuality operator is in fact an indexical form of historical necessity. Since there is no one length



Graph 2

the stick S would have in every possible continuation, they hold that if at t_0 the sentence "At t the stick S has the length the stick S actually has at t" expresses any proposition at all, it is a false one. As such, it cannot be the subject of knowledge, all the more knowledge a priori. The relevant formal analysis can be found in Section 6.2. of the Appendix.

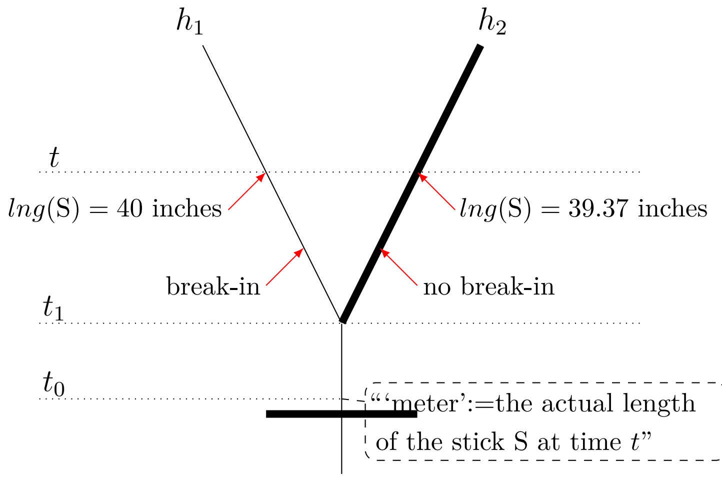
The question of the possibility of speaking about the actual future is closely related to the question of the possibility of ascribing truth values to propositions regarding contingent future. If we model indeterminism and the openness of the future using a tree model, this question can be formulated in the following way: Does any one branch of the tree play a special role in the semantic theory? This highlighted future is usually called, after Belnap and Green (1994), the Thin Red Line (TRL). Belnap himself is a staunch opponent of this idea, which has in turn been long defended by Danish philosopher Peter Øhrstrøm. Øhrstrøm claims that there is no contradiction between the indeterministic plurality of possible futures and the actual future course of events. The debate between Belnap and Øhrstrøm is to a large extent formal and revolves around the attempt to show whether branching time theory allows for the creation of a semantics that would use the notion of actual future to interpret sentences about the future. A concise description of this debate with a detailed bibliography can be found in (Wawer 2014).

In my recent work (Malpass, Wawer 2012; Wawer 2014) I have argued, in the spirit of Øhrstrøm's thought, that one can consistently assume that sentences regarding contingent future possess logical value. However, in the latter article I acknowledged that this thesis is not metaphysically neutral. I claimed that in order to reasonably uphold the thesis about the bivalence of sentences about the future and at the same time to believe in the plurality of future possibilities, one must make two metaphysical assumptions: (a) actualism, that is, the thesis that among all possible states of our world only one is concretely realized, while the rest are either *abstracts* or *modes* of the world's existence; and (b) eternalism, that is, the thesis that our world is an object that is not only spatially but also temporally extended and that is has concrete temporal parts going beyond the present (today I think that the bivalence thesis only requires the first assumption, I discuss this issue at length in the sixth chapter of Wawer 2016).

I believe that assumption (a) (and possibly also (b)) allows one to maintain, in a very natural and philosophically justified way, the bivalence of sentences about the future in light of metaphysical indeterminism. In my article, I also proposed a formal semantics of a tempo-modal language that is free of the problems raised by Belnap (the theory is improved and extended in Wawer 2016). A theory is therefore possible which yields the bivalence of contingent sentences about the future and this theory can be used to argue that the sentence (A) uttered at time t_0 expresses a historically contingent proposition the truth of which is knowable a priori.

There is no need to deal with the formal details here (an interested reader is referred to Section 6.3. of the Appendix). In order to understand the content, it will be enough for us to graphically distinguish the actual course of events using an emboldened line (Graph 3). Once we have at our disposal the distinguished course of events, the reference of the word 'meter' and the logical value of sentences featuring this word at t_0 can be established without any difficulty. Of course, no one can know before time t precisely how long the stick S will be at t. However, the relevant notion is already well defined at t_0 and propositions featuring it possess one of the two truth values.[8] At this point, one may raise the question regarding the utility of a notion whose conditions of correct use are unknown to the user and which the user will be able to use correctly only after time has revealed its meaning. However, these are pragmatic considerations. On the semantic level the example is perfectly correct.

[8] This situation formally resembles the instance where we hear the utterance "I am here" without knowing where the speaker is. Here too we know that the utterance expresses a true proposition, although we do not know precisely what proposition is being uttered, that is, we do not know who is where.



Graph 3

It turns out that (A) can be understood as a sentence expressing a historically contingent proposition whose truth is knowable a priori. However, the condition for doing so is that we agree for the ascription of truth values to propositions about contingent future and for the fixing of the reference of concepts by appealing to some future contingent states of affairs. This is also tied to the acceptance of a specific stance toward metaphysics of modality. As far as I can tell, these assumptions are not inconsistent or otherwise undesirable. In fact, some fragments of Naming and Necessity seem to suggest that this view was close to Kripke himself – the view that among the many future possibilities only one is actually realized. One could quote the following passage as an example: "Hence there are thirty-six possible states of the pair of dice, as far as the numbers shown face-up are concerned, though only one of these states corresponds to the way the dice actually will come out" (Kripke 1980: 16). We should note, however, that on this reading of Kripke's thesis, the view, that there exist contingent a priori truths becomes more controversial. To defend it, one would have to prove reasonable the metaphysical assumptions cited above which constitute the necessary condition for the interpretation of the sentence (A) as expressing a contingent a priori proposition.

4.2. Defence by Definitional Change

If one has no wish to concede that it is reasonable to speak about actual contingent future, but one would nonetheless like to acknowledge that Kripke's observations capture an important phenomenon, they must try to somehow redefine apriority or contingency. I will propose two redefinitions that will allow me to show how we can speak of the contingent apriority of Kripke's examples in the context of historical modality. Both reformulations will require some temporal-modal acrobatics.

4.3. Contingent A-Priori-in-the-Future Truths

In Section 3 I argued that at time t_0 no one can establish a priori the precise length the stick S will have at t since this is still undetermined at t_0. The response to this objection I presented in Section 4.1. was that, although it is impossible to establish precisely how long, say in inches, the stick will be, it is possible to establish that the sentence (A) expresses a true proposition. This response raises skepticism on the part of some theoreticians because the content of (A) refers to what will actually take place, while it is controversial whether one can speak of the actual future in the context of indeterminism. If we are not entitled to do so, then the proposition expressed by (A) is not only not knowable a priori at t_0 but it expresses a false proposition.

Even if we agree with the above objection, we can still defend the thesis that (A) enjoys some special status. Let us observe that at t_0 it was possible to establish that, regardless of how the world might unfold, the sentence (A) would express a true proposition as of t. Although the sentence would express a different proposition depending on the course of events, it would nonetheless always express a true proposition in the context in which it would be uttered. Therefore, it may be said that at t0 the sentence (A) had the "a-priori-in-the-future" status, that is, that it was knowable a priori at t_0 that in the future the sentence would express a true proposition.[9]

Definition 2 (a-priori-in-the-future)**.** It is knowable a-priori-in-the-future that a sentence φ expresses a true proposition if and only if, regardless of the past and the future course of events, there is a time such that, from that time on, if the sentence φ is uttered, then that sentence will express a true proposition.

[9] Keith Donnellan (1977: 24) writes that a very similar notion was suggested to him in a conversation by Roger Albritton. Donnellan uses the following expression to characterize this notion: "(. . .) what we can know is that certain sentences, if and when we come to be in a position to use them, will express truths" (Donnellan, 1977: 25).

Let us now return to the contingency of the proposition expressed by (A). I have argued above that the proposition is not historically contingent now. However, it is natural to say that there was a time such that it was *then* contingent what the sentence (A) says *now*. In other words, insofar as the reference of the word 'meter' has been fixed, we may ask whether there was a time such that it was then possible that the stick S would, and that it would not, be one meter long at t. Of course, this was the case until t_1. Therefore, we can say *today* that at t_1 it was historically contingent that (A). Moreover, at t_1 it was a-priori-in-the-future that (A). To put it another way, there was a time in the past such that it was then contingent whether (A), yet it was knowable a priori that the sentence (A) would express a true proposition in the future. This is one sense that can be ascribed to Kripke's thesis about the existence of contingent a priori propositions in the context of historical possibility. Below I will paraphrase this thesis in a similar but more natural manner.

4.4. Previously Contingent A Priori Truths

In the preceding section I tried to defend the thesis about the existence of contingent a priori truths while maintaining the notion of historical contingency and by assuming the notion of apriority-in-the-future. However, another tactic is also possible: to maintain that the truth of the proposition expressed by (A) is *today* knowable a priori, while it is *previously-contingent*. In the context of the historical notion of possibility this new sense of contingency can be explicated in the following way:

Definition 3 (previously-contingent)**.** A proposition φ is previously-contingent if and only if it was once the case that it was historically possible that φ and that it was historically possible that not-φ.

It is easy to note that the proposition expressed by the sentence (A) today has precisely this property. It used to be possible that at t the stick S would have a slightly greater length than it actually came to have. The proposition expressed by (A) is thus previously-contingent. Some of Kripke's formulations show that this understanding of possibility and contingency was close to him (for example, in describing possible and contingent states of affairs, he systematically employs the past form "might have" and not the present form "might").

Let us now ask how we should understand apriority in order that it is *today* knowable a priori that (A) expresses a true proposition (even if this was not always knowable in the past). I think that the following understanding can be proposed:

Definition 4 (now-a-priori)**.** It is knowable now-a-priori that a sentence φ expresses a true proposition if and only if, regardless of the past course of events, from now on, whenever the sentence φ is uttered, a true proposition will be expressed.[10]

The sentence (A) satisfies definition 4. Had the stick been slightly longer at t, the sentence "At t the stick S is one meter long" would still express a true proposition today (of course, in language that would be used today, where the word 'meter' would not refer to our meter but to a slightly greater unit). We can thus establish without recourse to experience that, regardless of precisely how things are with S at t, from t on the sentence (A) does express a true proposition (although the proposition may change depending on the length of the stick S at t).

Therefore, insofar as we know the relevant linguistic convention, the sentence "On November 22, 1875 at noon the stick S is one meter long" expresses a now-a-priori truth. This truth is at the same time previously-contingent. This is the new sense in which (A) is contingent a priori. Namely, it is previously-contingent now-a-priori. I think that this sense comes closest to the spirit of Kripke's text.

5. Summary

The subject matter of my analysis was Kripke's thesis that some sentences express propositions which are at the same time contingent and knowable a priori. I decided to explore this thesis in light of the notion of possibility I dubbed historical. I tried to show that if we define a contingent proposition as one which is historically possible, while its negation is also historically possible, the example sentences expressing contingent a priori truths discussed in the literature become controversial. It turns out that they express propositions whose truth is knowable a priori, but they are historically necessary. On the other hand, when these propositions are still historically contingent, it is doubtful whether they are knowable a priori. It seems that

[10] Let us note that a kindred definition: "It is knowable now-a-priori that a sentence φ expresses a true proposition if and only if from now on, whenever the sentence φ is uttered, a true proposition will be expressed" is flawed since it renders knowable a priori all historical truths, including "The Battle of Tannenberg took place in 1410," for example.

this conflict is not accidental but stems from the nature of the notions of time, contingency, and apriority.

Having noted the conflict between historical contingency and apriority, I went on to find a way to restore Kripke's thesis to its former power. I considered two lines of defence against the objection I had formulated. First, I argued that we can find sentences expressing historically contingent a priori propositions as long as we agree that semantic value can be fixed by reference to future contingent states of affairs. Following that, I showed how to defend Kripke's thesis by redefining either the notion of apriority or that of contingency.

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6. Appendix

6.1. Branching-Time (*BT*) Language and Models

In order to precisely express the problems I have raised, I will use a language proposed in (Belnap et al. 2001; Belnap 2002). It is perfectly suited for my purposes since, on the one hand, it is sufficiently rich to express all relevant definitions, and on the other hand, it was created with an eye to an interpretation in indeterministic branching models.

To pose our problem, we need a language of modal-temporal predicate logic with identity, equipped with a definite description operator and a truth predicate. Its logical constants are: the standard sentential operators of conjunction $\wedge$ and negation $\neg$; the temporal sentential operators "It will be the case that" (F), "It was the case that" (P) and "At t it is the case that" (At_t); the sentential operator "It is possible that" ($\Diamond$); the indexical modal operator "It is actually the case that" (@); the indexical temporal operator "It is now the case that" (Now); the truth predicate (tr); and the definite description operator ι which in Belnap's approach is a nominalizing functor with a sentential argument (regarding this question Belnap follows Frege against Russell). The rules for the composition of terms and formulas are standard.

The extralogical symbols of our language are: individual variables $x_1, x_2, \ldots$ (I will denote the set of individual variables with the symbol var); predicate constants $P, Q, R, \ldots$, including the two-place predicate of identity $=$; individual constants $a, b, c, \ldots$, including the individual constant s denoting the stick S and the individual constant †; temporal constants $t_1, t_2, \ldots$ referring to instants; and functional constants $f, g, h, \ldots$, including the one-place functional constant lng referring to the function of length.

In order to build semantics for the thus defined language, we need to define the relevant notion of a model. The BT model is an ordered quintuple $\mathfrak{M} := \langle M, \leqslant, D, Inst, I \rangle$, where:

1. $M \neq \emptyset$ is a set of possible moments, where a moment is understood as an instantaneous possible state of the world;

2. $\leqslant$ is a partial order defined on M, fulfilling the additional conditions of:

   (a) no backward branching:

   $$\forall m_1, m_2, m_3 \in M((m_2 \leqslant m_1 \wedge m_3 \leqslant m_1) \Rightarrow (m_2 \leqslant m_3 \vee m_3 \leqslant m_2));$$

   (b) consistency:

   $$\forall m_1, m_2 \in M \exists m_3 \in M(m_3 \leqslant m_1 \wedge m_3 \leqslant m_2).$$

   This order can be understood as a tempo-modal relation "inevitably-earlier — possibly-later." The pair $\langle M, \leqslant \rangle$ can be graphically represented as a tree of possibilities which can branch into the future but never into the past. The maximal linearly ordered chains of this tree (the maximal possible courses of events) are called histories and are denoted with the letters $h_1, h_2, h_3, \ldots$, while the set of all histories is denoted with the symbol $Hist$.

3. $Inst$ is a partition of the set M satisfying the following conditions:

   (a) one-element intersections: $\forall h \in Hist \forall i \in Inst\ card(h \cap i) = 1$;

(b) order preservation:

$$\forall i_1, i_2, h_1, h_2((h_1 \cap i_1 < h_1 \cap i_2) \Rightarrow (h_2 \cap i_1 < h_2 \cap i_2))$$

$$\forall i_1, i_2, h_1, h_2((h_1 \cap i_1 = h_1 \cap i_2) \Rightarrow (h_2 \cap i_1 = h_2 \cap i_2)).$$

The elements of the set $Inst$ can be thought of as temporal instants. Graphically, they are lines running perpendicular to the direction in which the tree of possibilities is growing. Thanks to them we can temporally compare events taking place within different histories. Although the order $\langle M, \leqslant \rangle$ is not linear, the set $Inst$ is linearly ordered. Hence, it is not time but possible courses of events that branch out in the misleadingly named branching-time theory. The symbol i_m denotes the instant of time where the moment m belongs.

4. $D \neq \emptyset$ is the set of possible objects containing, among others, the stick S and a distinguished non-existent object † serving as reference for non-denoting definite descriptions.

5. I is an interpretation function mapping:

   - individual constants into D,
   - temporal constants into the set of instants $Inst$,
   - the set of n-place predicate symbols into the set $\mathcal{P}(M \times Hist \times D^n)$,
   - the set of n-place functional symbols into the set of functions of the form: $M \times Hist \times D^n \mapsto D$.

The valuation V is a function mapping individual variables var into the domain D. For any $d \in D$, $V[d/x]$ denotes a valuation such that $\forall y(y \neq x \Rightarrow V(y) = V[d/x](y))$ and $V[d/x](x) = d$. The context is any $m \in M$.

The truth and reference for our language are relativized to a pair $\langle \text{moment}, \text{history} \rangle$. This formal measure has been proposed by Prior (1967) and subsequently developed by Thomason (1970, 1984). In the BT semantic we always assume that for any given pair $\langle m, h \rangle$ it is the case that $m \in h$; in order to highlight this fact in the notation, we write m/h instead of $\langle m, h \rangle$.

The reference of the term t in the model $\mathfrak{M}$, in the context m_c, given the valuation V, on the pair m/h (symbolically $t^{\mathfrak{M}, m_c, V, m/h}$), is defined inductively in the following way:

1. for the individual variable x: $x^{\mathfrak{M}, m_c, V, m/h} = V(x)$;

2. for the individual constant a: $a^{\mathfrak{M},m_c,V,m/h} = I(a)$;

3. for the n-argument functional symbol f and the n-tuple of terms $t_1,\ldots,t_n$:
$f(t_1,\ldots,t_n)^{\mathfrak{M},m_c,V,m/h} = I(f)(m,h,t_1^{\mathfrak{M},m_c,V,m/h},\ldots,t_n^{\mathfrak{M},m_c,V,m/h})$;

4. for the definite description $\iota x(A)$:

$$\iota x(A)^{\mathfrak{M},m_c,V,m/h} = \begin{cases} \text{(a)} & \text{the sole object } d \in D \text{ such that} \\ & \mathfrak{M},m_c,V[d/x],m/h \models A, \\ & \text{if such an object exists;} \\ \text{(b)} & \dagger, \text{ otherwise} \end{cases}$$

The notion of satisfaction for the formula φ in the model $\mathfrak{M}$, in the context m_c, given the valuation V, on the pair m/h (symbolically $\mathfrak{M},m_c,V,m/h \models \varphi$) is defined *via* standard induction:

1. for an n-place predicate symbol P, $\mathfrak{M},m_c,V,m/h \models P(t_1,\ldots,t_n)$ iff the n-tuple $\langle m,h,t_1^{\mathfrak{M},m_c,V,m/h},\ldots,t_n^{\mathfrak{M},m_c,V,m/h}\rangle$ belongs to $I(P)$;

2. $\mathfrak{M},m_c,V,m/h \models t_1 = t_2$ iff $t_1^{\mathfrak{M},m_c,V,m/h} \approx t_n^{\mathfrak{M},m_c,V,m/h}$, where $\approx$ is the relation of identity;

3. $\mathfrak{M},m_c,V,m/h \models \neg\varphi$ iff it is not the case that $\mathfrak{M},m_c,V,m/h \models \varphi$;

4. $\mathfrak{M},m_c,V,m/h \models \varphi \wedge \psi$ iff $\mathfrak{M},m_c,V,m/h \models \varphi$ and $\mathfrak{M},m_c,V,m/h \models \psi$;

5. $\mathfrak{M},m_c,V,m/h \models F\varphi$ iff $\exists m' \in h(m' > m$ and $\mathfrak{M},m_c,V,m'/h \models \varphi)$. The parameter h thus acquires particular importance during the interpretation of temporal operators. The truth of sentences of the form "It will be the case that φ" significantly depends on the choice of the history h used to interpret the sentence;

6. $\mathfrak{M},m_c,V,m/h \models P\varphi$ iff $\exists m' \in h(m' < m$ and $\mathfrak{M},m_c,V,m'/h \models \varphi)$;

7. $\mathfrak{M},m_c,V,m/h \models At_t\varphi$ iff $\mathfrak{M},m_c,V,m_{th}/h \models \varphi$, where m_{th} is the only moment in the history h belonging to the instant designated by the constant t (the definition of $Inst$ guarantees that such a moment exists);

8. $\mathfrak{M}, m_c, V, m/h \models \Diamond\varphi$ iff $\exists h' m \in h'$ and $\mathfrak{M}, m_c, V, m/h' \models \varphi$. This definition reveals the "historical" aspect of our notion of possibility. In order to establish whether at the moment m it is possible that φ, we must check only the histories that run through the moment m;

9. $\mathfrak{M}, m_c, V, m/h \models @\varphi$ iff $\forall h'(m_c \in h' \Rightarrow \mathfrak{M}, m_c, V, m_c/h' \models \varphi$. @ is an occasional expression. This is evidenced by the fact that the evaluation moment m on the left side of the equivalence has been replaced by the context m_c on the right side. This is a natural adaptation of David Kaplan's formalism (1989b). As I have written above, in the standard BT formalism, actuality is an occasional form of necessity (there is no one "actual" future), which is why we quantify over all histories running through m_c;

10. $\mathfrak{M}, m_c, V, m/h \models Now(\varphi)$ if and only if $\mathfrak{M}, m_c, V, (h \cap imc)/h \models \varphi$. "$Now$" is another occasional expression. In the case of its use, the evaluation history remains unchanged, while the moment of evaluation changes to the moment concurrent with the context of utterance. Precisely this behaviour of the operator @ is required in order that sentences of the type "I could be living in London now" be properly interpreted;

11. $\mathfrak{M}, m_c, V, m/h \models tr`\varphi'$ iff $\mathfrak{M}, m, V, m/h \models \varphi$, where '$\varphi$' is the name of the sentence φ. The key feature of the thus functioning predicate is that it replaces the context of utterance with the current moment of evaluation. Owing to this, we evaluate sentences in circumstances in which they are uttered and so, for example, the sentence "The children are quiet now" uttered an hour ago was true insofar as the children were quiet an hour ago and not insofar as they are quiet now.

The remaining extensional sentential connectives are defined in the standard way; each modal operator is associated with a dual operator: $\Box = \neg\Diamond\neg$ ("It is settled that"); $G = \neg F \neg$ ("It will always be the case that"); and $H = \neg P \neg$ ("It was always the case that").

6.2. Indeterminism, Actual Future and the Conflict of Apriority and Historical Necessity

Equipped with this rich formal apparatus we are finally able to model our example. May lng^* be a length function mapping objects to their lengths, and may s be the constant designating the stick S ($I(lng) = lng^*$, $I(s)$ =S). We can now define the length of one meter (m) in the following way:

Definition 5 (meter). $m := \iota x @ At_t(x = lng(s))$.

Hence, one meter is the distance equivalent to the actual length of the stick S at time t. Given this definition, the sentence (A) has the following logical form:

(A) $At_t(lng(s) = m)$, that is, $At_t(lng(s) = \iota x @ At_t(x = lng(s)))$.

Hence, Kripke's example says that at time t the length of the stick S is equal to the length the stick S actually has at time t. Let us first distinguish this sentence from another similar one:

(A*) $At_t(lng(s) = \iota x At_t(x = lng(s)))$.

This is simply a tautology and thus expresses a necessary proposition. In order to confirm that, I will go through a tedious but simple process of establishing the truth conditions for this example:

1. $\mathfrak{M}, m_c, V, m/h \models At_t(lng(s) = \iota x At_t(x = lng(s)))$ iff (def. At_t)
2. $\mathfrak{M}, m_c, V, m_{th}/h \models lng(s) = \iota x At_t(x = lng(s))$ iff (def. =)
3. $lng(s)^{\mathfrak{M},m_c,V,m/h} \approx \iota x At_t(x = lng(s))^{\mathfrak{M},m_c,V,m/h}$ iff (def. ι and $\mathfrak{M}$)
4. $lng^*(m_{th}, h, S) \approx$ the sole object $d \in D$ such that $\mathfrak{M}, m_c, V[d/x]$, $m_{th}/h \models At_t(x = lng(s))$ iff (def. At_t)
5. $lng^*(m_{th}, h, S) \approx$ the sole object $d \in D$ such that $\mathfrak{M}, m_c, V[d/x]$, $m_{th}/h \models x = lng(s)$ iff (def. =)
6. $lng^*(m_{th}, h, S) \approx$ the sole object $d \in D$ such that $x^{\mathfrak{M},m_c,V[d/x],m_{th}/h}$ iff (def. $\mathfrak{M}$)
7. $lng^*(m_{th}, h, S) \approx$ the sole object $d \in D$ such that $d \approx lng^*(m_{th}, h, S)$.

Since lng^* is a function, it is guaranteed that there exists precisely one such magnitude d, and therefore, that our sentence (A^*) says that $d = d$, which is obviously a necessary proposition.

If the truth conditions for (A) are analyzed equally precisely, it will turn out that it is true at the point $\mathfrak{M}, m_c, V, m/h$ if and only if:

$$\mathbf{A} : lng^*(m_{th}, h, S) \approx \text{the sole object } d \in D \text{ such that } \forall h'(m_c \in h' \Rightarrow d \approx lng^*(m'_{th}, h', S)).$$

The difference in truth conditions between (A*) and (A) is therefore fundamental. Whereas the first sentence expresses a logically necessary proposition, the second one is true in certain circumstances, if the length of the stick S in these circumstances at time t is equal to the length the stick has at t in actual circumstances (where actual circumstances of the given utterance are understood to be the bundle of histories running through the context of this utterance). Let us now take a look at the simplified branching model presented graphically in the article, with only two histories h_1 and h_2 branching out at time t_1. In both histories, the stick S is placed at Sèvres at time t, although it has in each case a slightly different length: 40 inches in the case of h_1 and 39.37 inches in the case of h_2. Let us denote this model with the symbol $\aleph$. If we look at it carefully, it will turn out that the equivalence **A** introduced above is true if and only if $i_{mc} > I(t_1)$, that is to say, for contexts belonging to instants later than t_1. Otherwise, the equivalence does not obtain since its left side always yields a positive magnitude (40 or 39.37 inches depending on the history chosen), while the right side always yields †; this is because there is no magnitude which would be equal to the length of the stick S in every history running through m_c – there is no magnitude which would be simultaneously equal 39.37 and 40 inches. It thus turns out that the sentence (A) uttered at any time after t_1 expresses a true proposition, while at t_1 and earlier it expresses a false proposition.

It is therefore obvious that the proposition expressed by the sentence (A) before t_1 cannot be known a priori since it is not even true. The sentence (A) can thus only express an a priori truth insofar as it is uttered at times later than t_1.

Let us now consider the modal status of the proposition expressed by the sentence (A) at times later than t_1. First, the definition of historical contingency ($\diamondsuit\!\!\!\cdot$):

Definition 6 (historically contingent)**.** $\diamondsuit\!\!\!\cdot\,\varphi :\Leftrightarrow \Diamond\varphi \wedge \Diamond\neg\varphi$.

It is already clear form our graph that if we choose some moment m_c belonging to the history h_1 and to any time $t_x > t_1$, the proposition expressed by (A) will be a historically necessary proposition. Regardless of which history running through m_c we choose, $lng^*(m_{th}, h, s^{\aleph, m_c, m_{th}/h})$ will equal 40 inches, and the sole object which is the length of the stick S in each history h such that $m_c \in h$ will be 40 inches. Hence, in any history such that $m_c \in h$ and m_c belongs to a time later than t_1, the sentence (A) expresses the proposition that 40 inches $\approx$ 40 inches, which is obviously a necessary proposition. Analogously, if we choose $m_c \in h_2$ and $m_c \in t_x > t_1$, the sentence (A) will express this necessary proposition: 39.37 inches $\approx$ 39.37 inches.

It thus turns out that, as long as the length of the stick S at time t is not determined, the sentence (A) will express a false proposition, and therefore one not knowable a priori, while once the course of events has determined the length of S at t, the sentence (A) will express a true proposition knowable a priori, but a necessary one. This observation is generalizable over any BT model:

Fact 1. $\forall \mathfrak{M} \forall m_c \forall V \forall h(\mathfrak{M}, m_c, V, m_c/h \models A$ iff $\mathfrak{M}, m_c, V, m_c/h \models \Box A)$.

6.3. Solution 1: Historically Contingent A Priori Propositions

The idea I presented in section 4.1. was to maintain that the sentence (A) expresses a proposition whose truth is knowable a priori before time t_1, when the length the stick S is still contingent. However, the BT formalism presented above yielded the result that, at such times, the sentence (A) expresses a necessarily false proposition. Therefore, in order to verbalize this idea, we must modify our semantics, or more precisely, we must change the definition of the operator @. Intuitively speaking, we want to tie the interpretation of @ with the possible history that will actually be realized. The information contained in the notion of a model presented so far is not sufficient to distinguish one history as the "actual course of the world." Therefore, for the sake of formal analysis, we will distinguish one history representing the actual course of events and call it the Thin Red Line (TRL). Now the model has the form $\mathfrak{M} := \langle M, TRL, \leqslant, D, Inst, I \rangle$, where $TRL \in Hist$. We must additionally assume that for any m_c, $m_c \in TRL$ (I justify this assumption in detail in (Wawer 2014)). Despite this change, the definitions of the reference of terms and of satisfaction for connectives remain the same. The only functor that makes significant use of the TRL parameter is the functor "It is actually the case that":

- $\mathfrak{M}, m_c, V, m/h \models @^*\varphi$ iff $\mathfrak{M}, m_c, V, m_c/TRL \models \varphi$.

Therefore, it is true that φ is actually the case insofar as φ is true in the history that actually obtains (the *TRL* history). In light of this definition, the sentence (A) has the form

$$At_t(lng(s) = \iota x @^* At_t(x = lng(s)))$$

and is true at the point $\mathfrak{M}, m_c, V, m/h$ insofar as:

$$lng * (m_{th}, h, S) \approx \text{ the sole object } d \text{ such that } d \approx lng^*(m_{tTRL}, TRL, S).$$

In other words, (A) is true in the history h insofar as at time t in this history the stick S has the same length that it has at t in the actual history.

Due to the change made in the notion of a model and the definition of @, the expression "one meter" is well defined regardless of circumstances and refers to one and the same length, that is to say, the length the stick S has at time t in the actual course of events.'

Given this definition, it turns out that in the model ℵ, in which we assume h_2 to be the *TRL*, at any moment before time t_1 the sentence (A) expresses a contingent truth since in the history h_2 at time t the stick has the same length it has in the *TRL*, while its length in the history h_1 is different.

Fact 2. $\exists\mathfrak{M}\exists m_c \exists V(\mathfrak{M}, m_c, V, m_c/TRL \models \Diamond A$ and $\mathfrak{M}, m_c, V, m_c/TRL \models \Diamond\neg A)$.

We have yet to confirm whether the truth of the proposition expressed by the sentence (A) is knowable a priori before the moment t_1. In order to give formal expression to the intuitions discussed in this text, I propose the following definition that should render more precise what it means that the truth of the proposition expressed by a sentence φ is knowable a priori (◯φ):

Definition 7 (a priori)**.** $\bigcirc\varphi \Leftrightarrow H\Box G(uttered\text{‘}\varphi\text{’} \rightarrow tr^*\text{‘}\varphi\text{’})$,

where *uttered* is a one-place predicate true of φ in the given circumstances if and only if φ is uttered in these circumstances, while tr^* is the following modification of the predicate *tr*:

- $\mathfrak{M}, m_c, V, m/h \models tr^*\text{‘}\varphi\text{’}$ iff $\mathfrak{M}', m, V, m/h \models \varphi$, where $\mathfrak{M}'$ differs from $\mathfrak{M}$ only in that the *TRL* in the model $\mathfrak{M}'$ is the history h.

Therefore, we may say that the truth of the proposition expressed by φ is knowable a priori if and only if in any temporal and modal circumstances it is the case that the sheer fact of uttering φ guarantees the truth of the proposition expressed by it. In this sense, it is for example knowable a priori that the sentence "I am here now" expresses a true proposition.

It is easy to find that in the context of our narrative modelled by ℵ the truth of the proposition expressed by (A) is knowable a priori since the following condition is satisfied:

$$\forall m_c \forall m \forall h \aleph, m_c, V, m/h \models uttered\text{‘}A\text{’} \rightarrow tr^{*}\text{‘}A\text{’}.$$

I leave it to an attentive reader to check the correctness of this implication. I will only point out that the key reason why the condition is satisfied by the sentence (A) is that the operator @ is each time tied to one selected history and not a bundle of histories, as was the case in Belnap's (2001) and MacFarlane's (2008) definitions. We can interpret the operator @ the way we do because we have introduced into the definition of the model an additional element: the actual history. Therefore, in keeping with what I held in section 4.1., if we agree to this conceptual move and its attendant philosophical assumptions, we will be able to show that in certain contexts the sentence (A) expresses a historically contingent proposition whose truth is knowable a priori.

6.4. Solution 2: Defence by Definitional Change

Those not willing to admit that we can reasonably speak of the actual future in the context of an indeterministic world, but sympathetic to Kripke's considerations, may defend his thesis by redefining either the notion of apriority or that of contingency.

6.4.1. Contingent and A-Priori-in-the-Future

Let us formally consider the operator a-priori-in-the-future (⧋) discussed above (section 4.3.):

Definition 8 (a-priori-in-the-future)**.** ⧋φ ↔ $H\Box FG$(‘φ’ *is uttered* → *tr*‘φ’).

Intuitively: in every possible course of events there will be a time such that from that time on if the sentence φ is uttered, it will be uttered truly.

With this notion at our disposal we can argue, as I have in this text, that insofar as the precise length of the stick S at time t is determined, we can say that there was a time in the past such that it was then historically contingent that (A) and that (A) was then a-priori-in-the-future. Symbolically:

Fact 3. $\forall t_x > t_1 \forall m \in t_x \forall V \in h(m \in h \Rightarrow \aleph, m, V, m/h \models P(\Diamond\!\!\!\!\text{c}\; A \wedge \bigcirc\!\!\!\!\!\triangle\, A).$

Let us sketch the proof. In order to assess the truth of the above fact, let us take any instant $t_x > t_1$ and any moment $m \in h$ at this instant. We must check whether there exists a moment $m' < m$ such that:

1. $\aleph, m, V, m'/h \models \Diamond At_t(lng(s) = \iota x @At_t(x = lng(s)))$

2. $\aleph, m, V, m'/h \models \Diamond\neg At_t(lng(s) = \iota x @At_t(x = lng(s)))$

3. $\aleph, m, V, m'/h \models A$

It turns out that the moment $m' \in t_1$ satisfies these conditions. Let us focus on condition 1 first. $\aleph, m, V, m'/h \models \Diamond At_t(lng(s) = \iota x @Att(x = lng(s)))$ if and only if $\exists h' \aleph, m, V, m'/h' \models At_t(lng(s) = \iota x @Att(x = lng(s)))$. In other words, we must check whether there exists a history running through m' such that the length of the stick S in this history is the same as the length the stick actually has (relative to the moment m) at time t (I call it to the reader's attention that here we are using the operator @ and not @*). Of course, every history running through m will be evidence of that. Now, let us take a look at condition 2. We must check whether there exists a history running through m' in which the length of the stick S is different than it actually is, that is, different than in all the histories running through m. Our graph confirms that this is the case. Regardless of which history we refer to for the moment $m \in t_x$, the length of the stick S is already determined in this history (because $t_x > t_1$); moreover, there exists another history running through m' in which the length is different. Hence, the contingency criteria are satisfied. Finally, let us consider condition 3. We need to investigate whether in every history $h \in Hist$ there is a moment $m_1 \in h$ such that for each $m_2 > m_1$, $\aleph, m, V, m_2/h \models (\text{'}A\text{'}\ \textit{is uttered} \Rightarrow tr\text{'}A\text{'})$. This implication is true at any $m \in t$ and so condition 3 is satisfied. As a result, we have the right to draw the conclusion that at any moment at which the length of the stick S is determined it is the case that it was once contingent and a-priori-in-the-future.

6.4.2. Previously-Contingent Now-A-Priori

I will now try to recreate the most natural understanding of Kripke's thesis I discussed in section 4.4. First, I will introduce the notion of previous-contingency ($\Diamonddot$):

Definition 9 (previously-contingent)**.** $\Diamonddot\varphi \leftrightarrow P(\Diamond\varphi \wedge \Diamond\neg\varphi)$.

Followed by the notion of now-apriority $\textcircled{\triangledown}$:

Definition 10 (now-a-priori)**.** $\textcircled{\triangledown}\varphi \rightarrow H\Box NowG(\text{`}\varphi\text{'}\ is\ uttered \rightarrow tr\text{`}\varphi\text{'})$.

Intuitively: Regardless of what the past was like and what the future will be like, from now on, if one were to utter the sentence φ, one would utter it truly.
This notion of apriority is sensitive to the moment of utterance. For some sentences, it is the case that their truth is knowable a priori at certain moments, while at some other moments it is not. The sentence (A) in our model is a perfect example of this. It turns out that:

Fact 4. $\forall i \in Inst \forall m \in i \aleph, m, V, m/h \models \textcircled{\triangledown}A$ iff $i > I(t_1)$.

Fact 4 says that until time t_1 the sentence (A) did not express a proposition whose truth could be established a priori, but it has since time t_1. At the same time, let us note that:

Fact 5. $\forall i \in Inst \forall m \in i \aleph, m, V, m/h \models \Box A$ iff $i > I(t_1)$.

And thus:

Fact 6. $\forall i \in Inst \forall m \in i \aleph, m, V, m/h \models \textcircled{\triangledown}A$ iff $m, V, m/h \models \Box A$.

The last fact can serve as a formal expression of the conflict between the notion of historical apriority and that of historical contingency I pointed out at the beginning of the article. Nonetheless, if we adopt the generalized notion of previous-contingency, the conflict can easily be avoided by noting that:

Fact 7. $\forall i \in Inst \forall m \in i(i > I(t_1) \Rightarrow \aleph, m, V, m/h \models \textcircled{\triangledown}A$ and $\aleph, m, V, m/h \models \Diamonddot A)$.

This formulation is perhaps the closest formal analog of Kripke's thesis regarding the existence of contingent propositions whose truth is knowable a priori.

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